MORE ON NEUTRINO OSCILLATION PHYSICS WITH A NEUTRINO FACTORY.

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EVIDENCE FOR ν OSCILLATION

SuperK $\quad \nu_\mu \to \nu_x \quad x \neq e$

90% c.l. $\quad \sin^2 2\theta > 0.8 \qquad 5 \cdot 10^{-4} < \Delta m^2 < 6\ 10^{-3}\ eV^2$

LBL Fermilab-Soudan, CERN-GS. incapable of exploring the whole allowed region.

Solar $\quad \bar{\nu}_e \to \bar{\nu}_x \quad$ MSW or vacuum osc.

$$\Delta m^2 \begin{cases} 10^{-5} - 10^{-4}\ eV^2 \\ 10^{-10}\ eV^2 \end{cases}$$

Current terrestial experiments have no direct access to the solar mass range.

Solar ⊕ Superk easily accommodated in the theoretically appealing three family mixing scenario

$$\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$$

THREE-FAMILY MIXING

	# angles	# phases
Dirac Fermions	$\frac{N(N-1)}{2} = 3$	$\frac{(N-2)(N-1)}{2} = 1$

$$\begin{pmatrix} V_{\nu_e e} & V_{\nu_e \mu} & V_{\nu_e \tau} \\ V_{\nu_\mu e} & V_{\nu_\mu \mu} & V_{\nu_\mu \tau} \\ V_{\nu_\tau e} & V_{\nu_\tau \mu} & V_{\nu_\tau \tau} \end{pmatrix} = \begin{pmatrix} c_{12} c_{13} & c_{13} s_{12} & s_{13} \\ -c_{23} s_{12} e^{i\delta} - c_{12} s_{13} s_{23} & c_{12} c_{23} e^{i\delta} - s_{12} s_{13} s_{23} & c_{13} s_{23} \\ s_{23} s_{12} e^{i\delta} - c_{12} c_{23} s_{13} & -c_{12} s_{23} e^{i\delta} - c_{23} s_{12} s_{13} & c_{13} c_{23} \end{pmatrix}$$

	# angles	# phases
Majorana Fermions	$\frac{N(N-1)}{2} = 3$	$\frac{N(N-1)}{2} = 3$

$$MM. = CKM \begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}$$

Observables that depend on the Majorana phases are suppressed by factors

$$\frac{m_{\nu_i}}{E} \longrightarrow \text{observation is quite hopeless!}$$

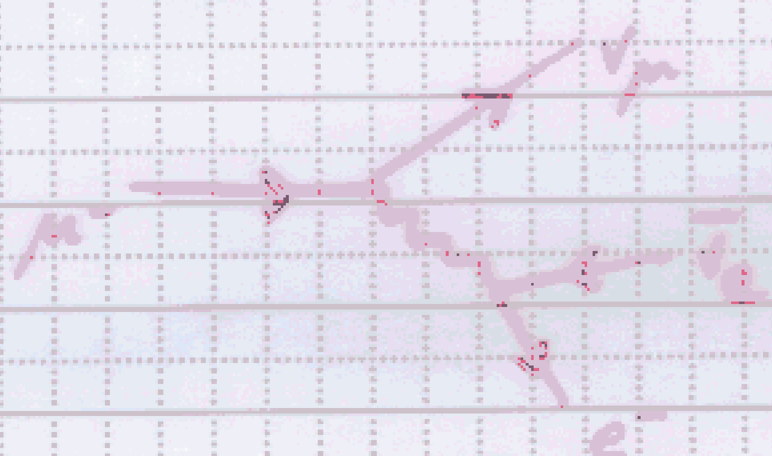
μ - accumulator as ν factory

per year   e.g

$n_\mu = 2 \cdot 10^{20}$ μ^+/μ^- per year $E_\mu = 20$ GeV

LBL = 732 km   CERN-GS
Fermilab-Soudan

Extremely well-known beam !   Geer



$$\left.\frac{dN_\nu}{dy\,dS}\right|_{\cos\theta=0} = \frac{E_\mu^2\, n_\mu}{\pi\, m_\mu^2\, L^2} F_\nu(y) \qquad y = \frac{E_\nu}{E_\mu}$$

$$F_{\nu_e,\bar{\nu}_e} = 12y^2(1-y)\,\theta(y)\,\theta(1-y)$$

$$F_{\nu_\mu \bar{\nu}_\mu} = 2y^2(3-2y)\,\theta(y)\,\theta(1-y)$$

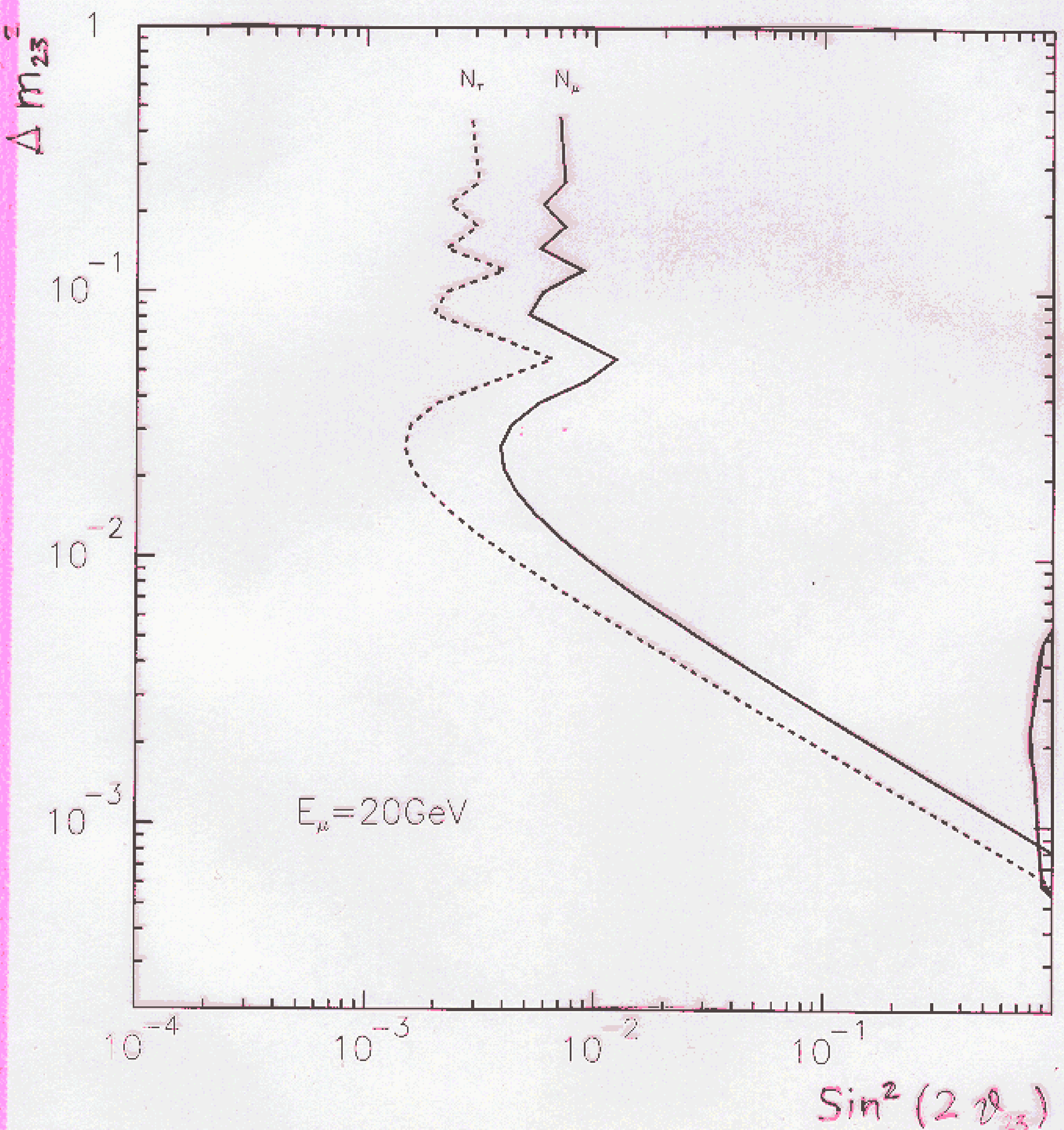
$$\#_{CC} \propto \frac{E_\mu^3}{L^2}$$

Two-family mixing

N_μ 10 kTon

N_τ 1 kTon (opera like 35% efficiency)

$\nu_\mu \leftrightarrow \nu_\tau$



Minimal Scheme : Solar ⊕ Atmospheric

$$|\Delta m_{32}^2| \gg |\Delta m_{21}^2|$$

Atmospheric osc.

Δm_{32}^2

θ_{23}

θ_{13}

Solar Osc.

Δm_{21}^2

θ_{12}

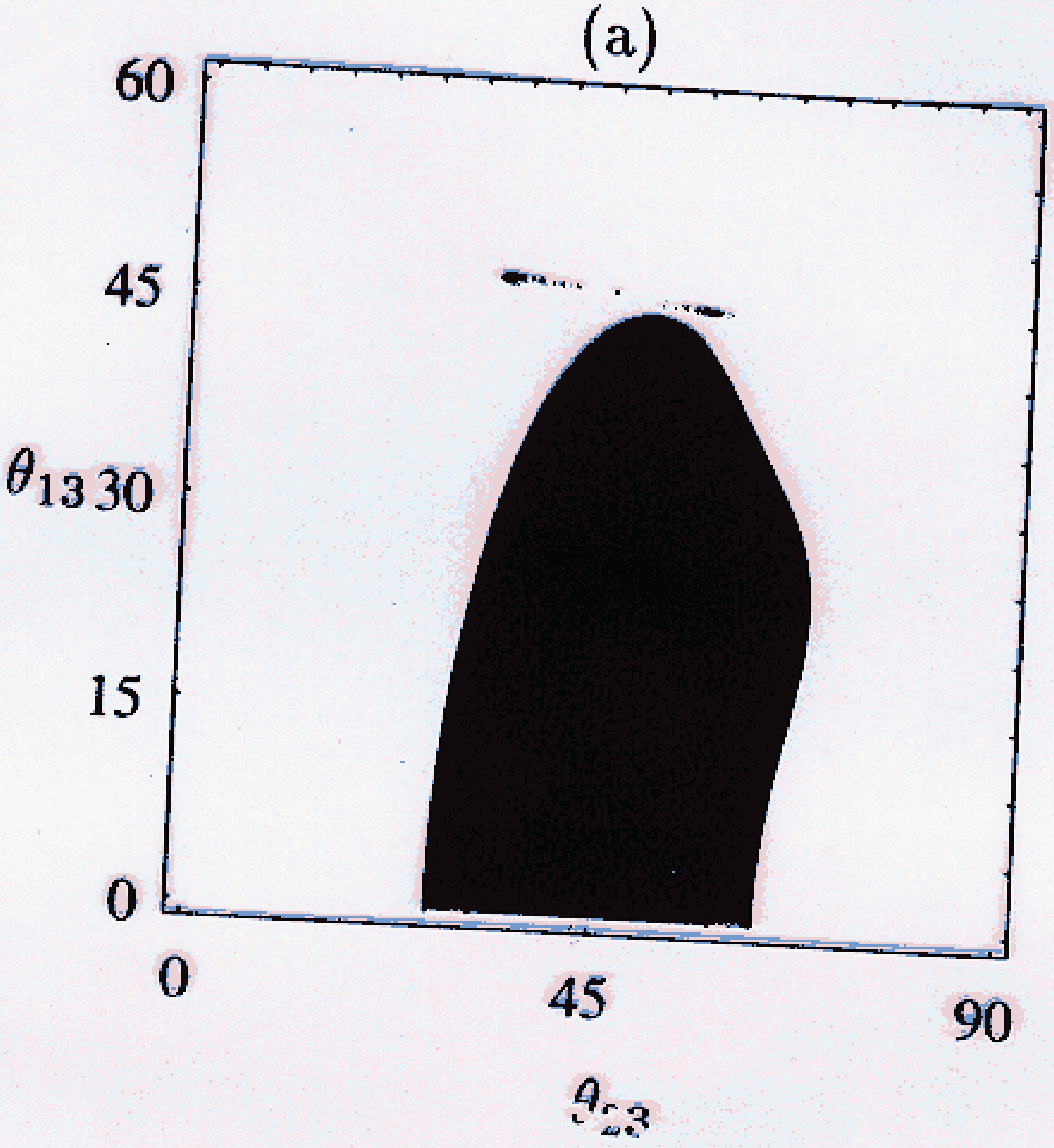
θ_{13}

$$P_{\nu_e\nu_\mu} = 4\, s_{23}^2\, s_{13}^2\, c_{13}^2 \sin^2 \frac{\Delta m_{32}^2 L}{4E}$$

$$P_{\nu_e\nu_\tau} = 4\, c_{23}^2\, s_{13}^2\, c_{13}^2 \sin^2 \frac{\Delta m_{23}^2 L}{4E}$$

$$P_{\nu_\mu\nu_\tau} = 4\, c_{13}^4\, s_{23}^2\, c_{23}^2 \sin^2 \frac{\Delta m_{32}^2 L}{4E}$$

μ - accumulator in principle has access to all these transitions.



SK Limit $\Delta m^2_{12} << 10^{-3}$

any ϑ_{12}

hep-ph/9807235 Barbieri et al.

Barbieri et al.

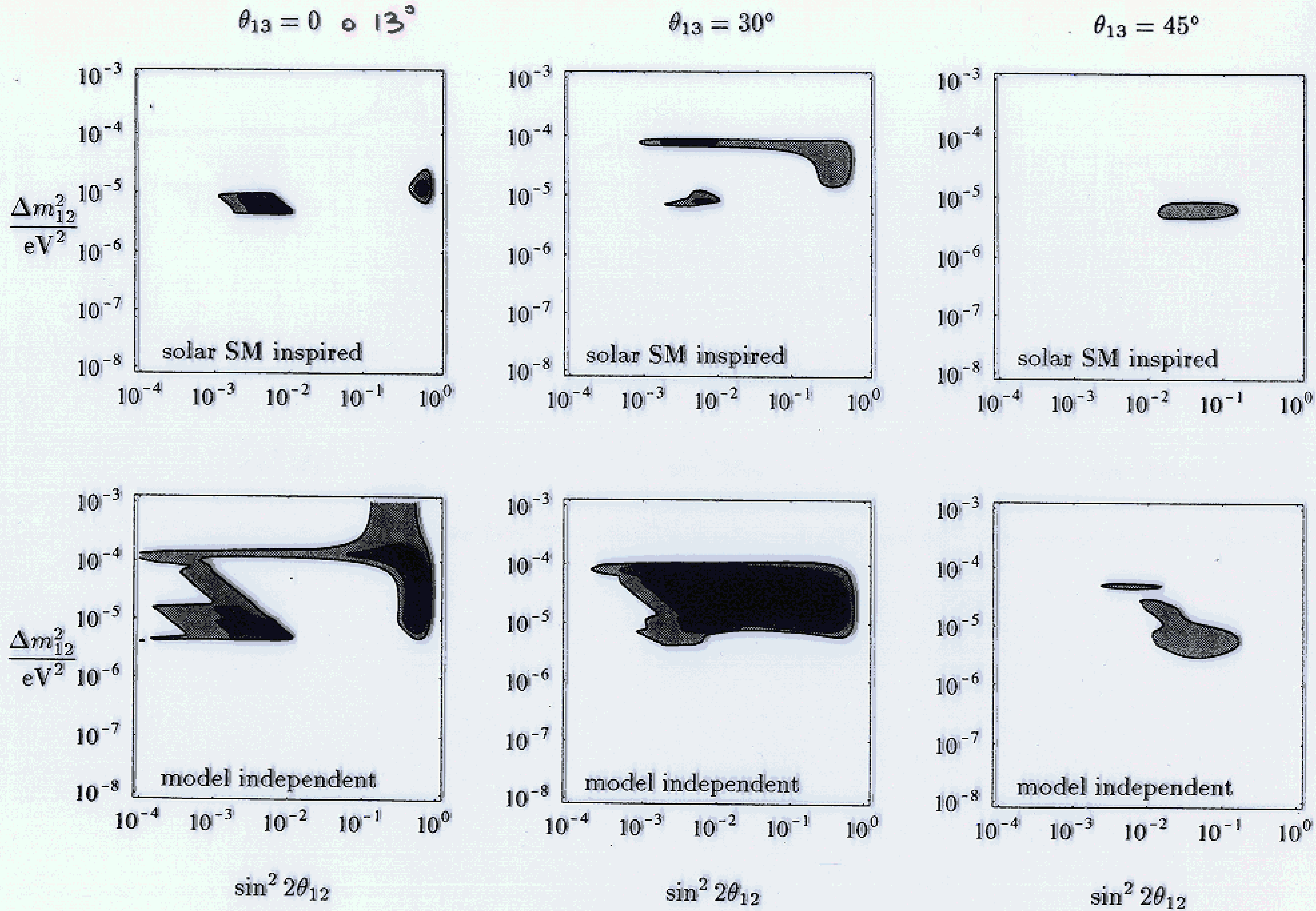


Figure 3: *Allowed regions in the plane* $(\sin^2 2\theta_{12}, \Delta m^2_{12})$ *for* $\theta_{13} = 0, 30°$ *and* $45°$. *The upper plots assume that*

$$\mu^- \rightarrow \nu_\mu \bar{\nu}_e e^-$$

	stat. error
$\bar{\nu}_e \rightarrow e^+$	$1/\sqrt{N}$
$\bar{\nu}_\mu \rightarrow \mu^+$	$1/N$
$\bar{\nu}_\tau \rightarrow \tau^+$	$1/N$
$\nu_e \rightarrow e^-$	$1/N$
$\nu_\mu \rightarrow \mu^-$	$1/\sqrt{N}$
$\nu_\tau \rightarrow \tau^-$	$1/N$

Search for wrong-sign μ

Extremely sensitive appearance measurement

Setup 10 kTon detector (Minos like?)

μ^+/μ^- and μ/other separation

$E_\nu > 5$ GeV only statistical errors!

Backgrounds (homework for experimentalist in WG)

NC contamination (π, K decay)

charm

charge missid.

732 km

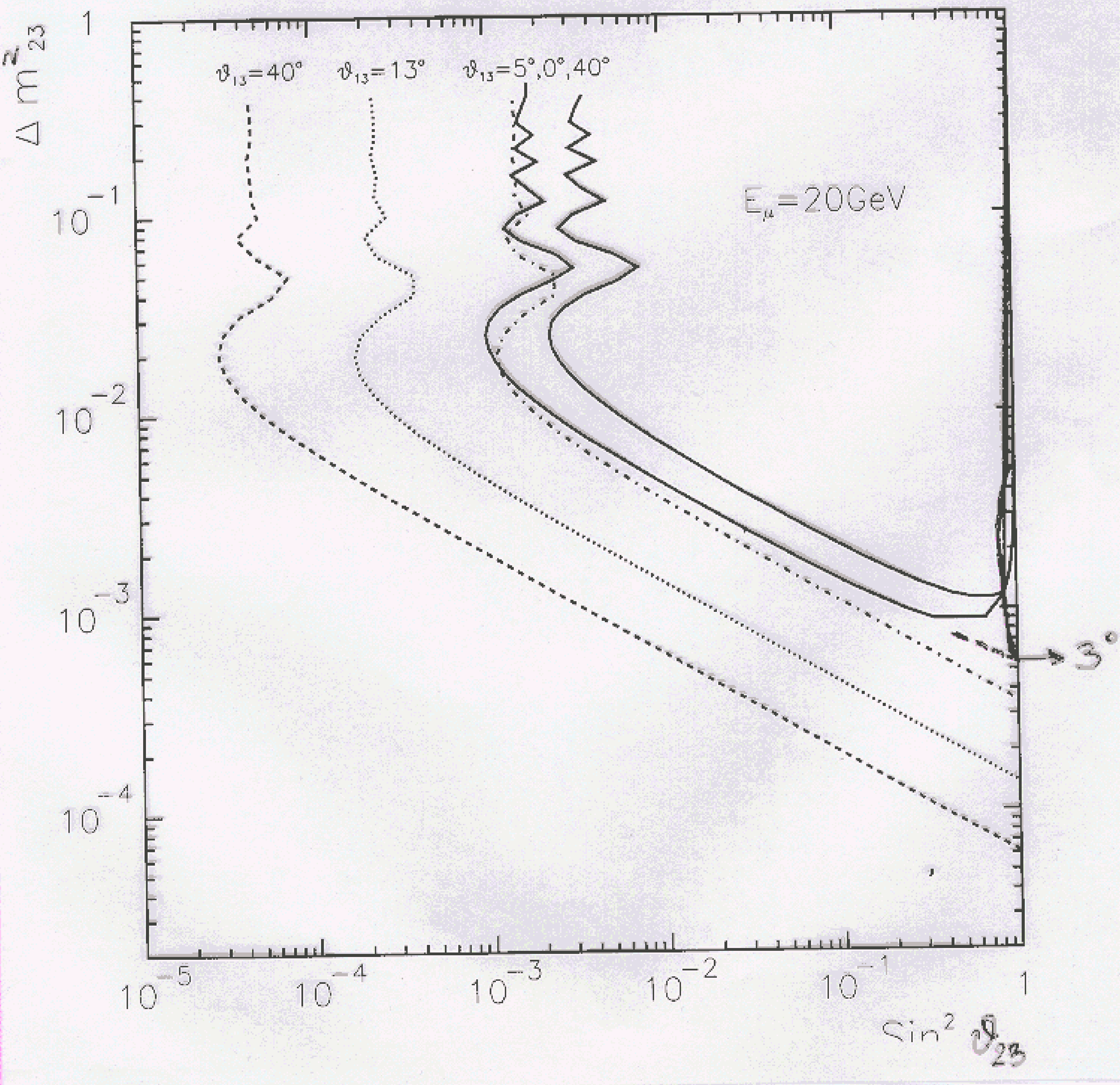
$< 10^{-4} - 10^{-5}$

$\sin^2\theta_{13}$? V_{e3} vs V_{ub} ? $3°$?

Three-family mixing

10 kTon $E_\nu > 5$ GeV

only statistical errors!

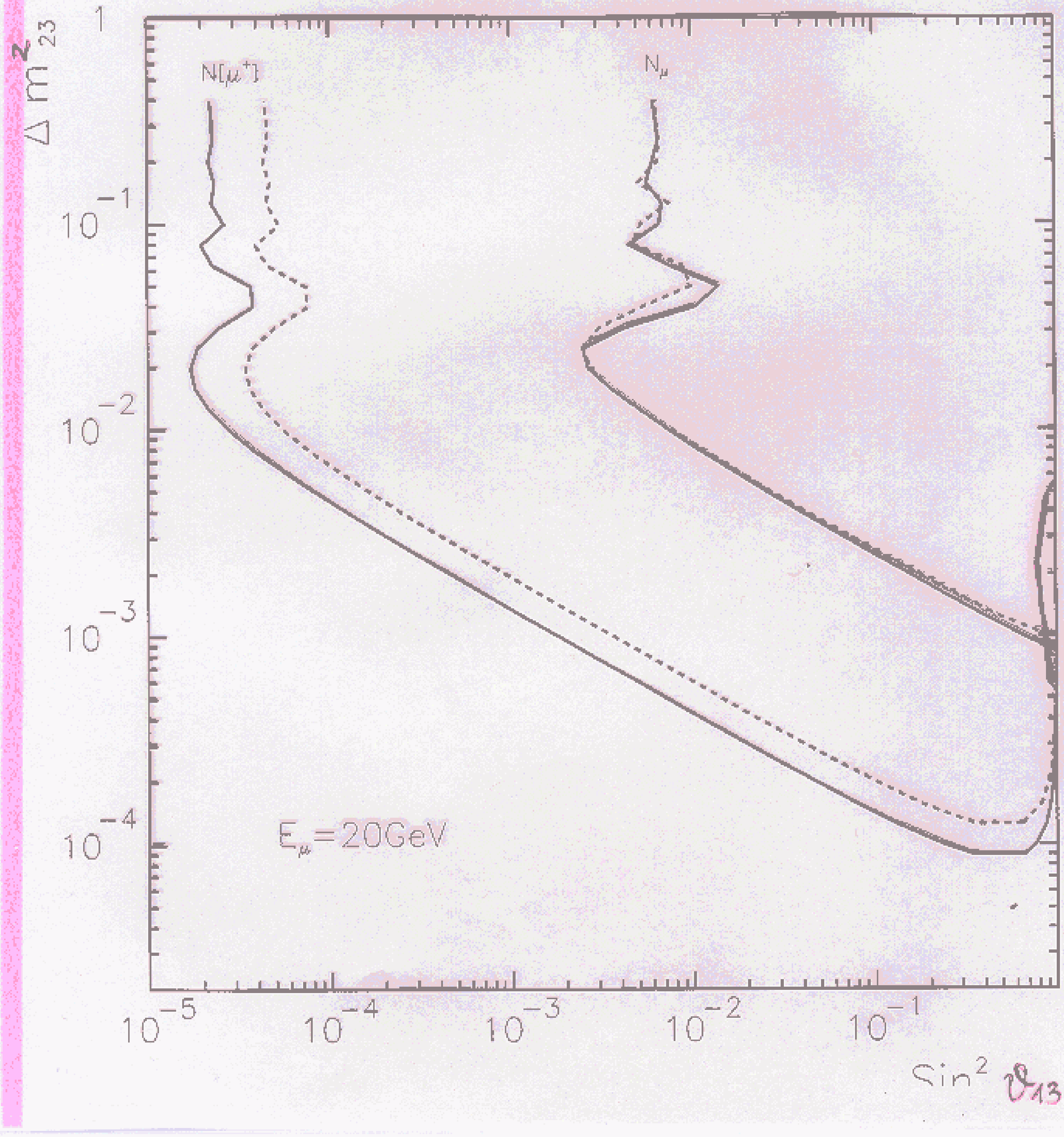


Three-family mixing

10 kTon

Only statistical errors!

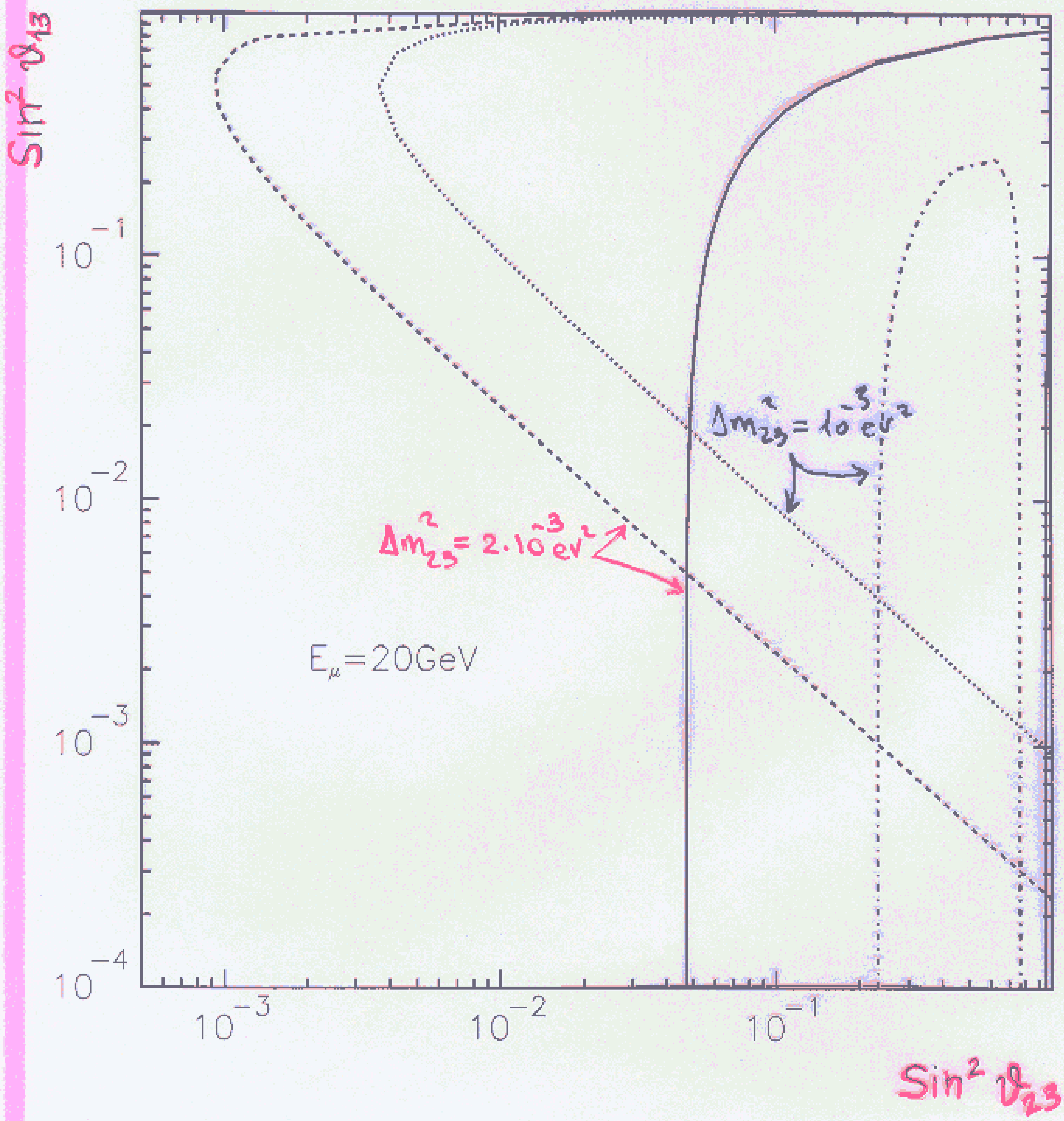
$E_\nu > 5$ GeV



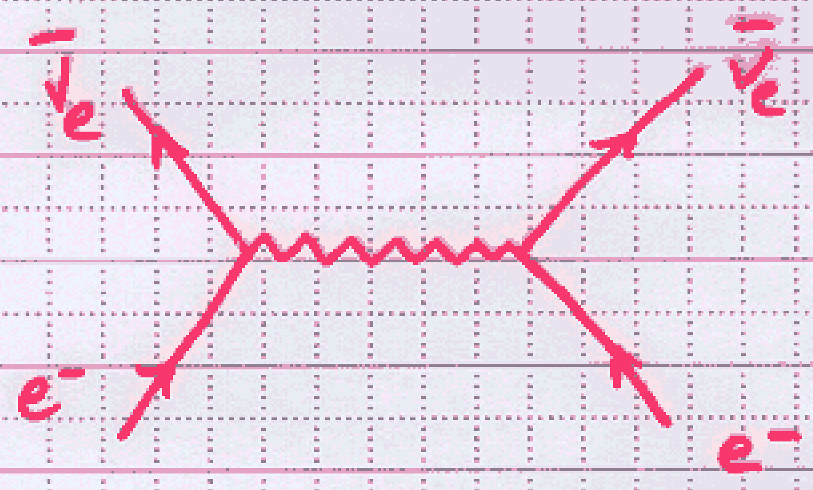
Three-family mixing

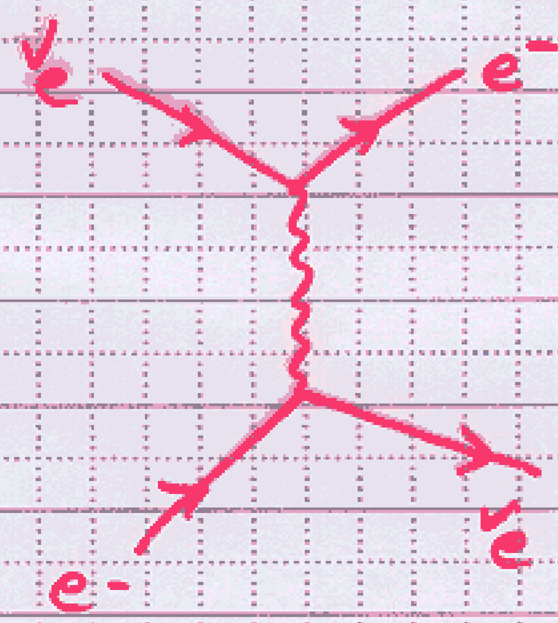
10kTon
Only statistical errors!

$E_\nu > 5$ GeV



Matter Effects





$$\Delta m^2_{\nu_e} = \pm 2 E_\nu A \qquad A = \sqrt{2}\, G_F\, n_e$$

$$M^{matter} = V \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} V^+ + \begin{pmatrix} \overbrace{\pm 2 E_\nu A}^{\pm \mu} & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

$$= \tilde{V} \begin{pmatrix} \tilde{m}_1^2 & & \\ & \tilde{m}_2^2 & \\ & & \tilde{m}_3^2 \end{pmatrix} \tilde{V}^+$$

$A \simeq 10^{-13}$ eV $\quad E_\nu \sim 12$ GeV

~~quote $\rho \sim 1.8$ gr/cm^3~~

($\rho \sim 2.8$ gr/cm^3)

$$\boxed{\mu = 2.4 \cdot 10^{-3}\, eV^2}$$

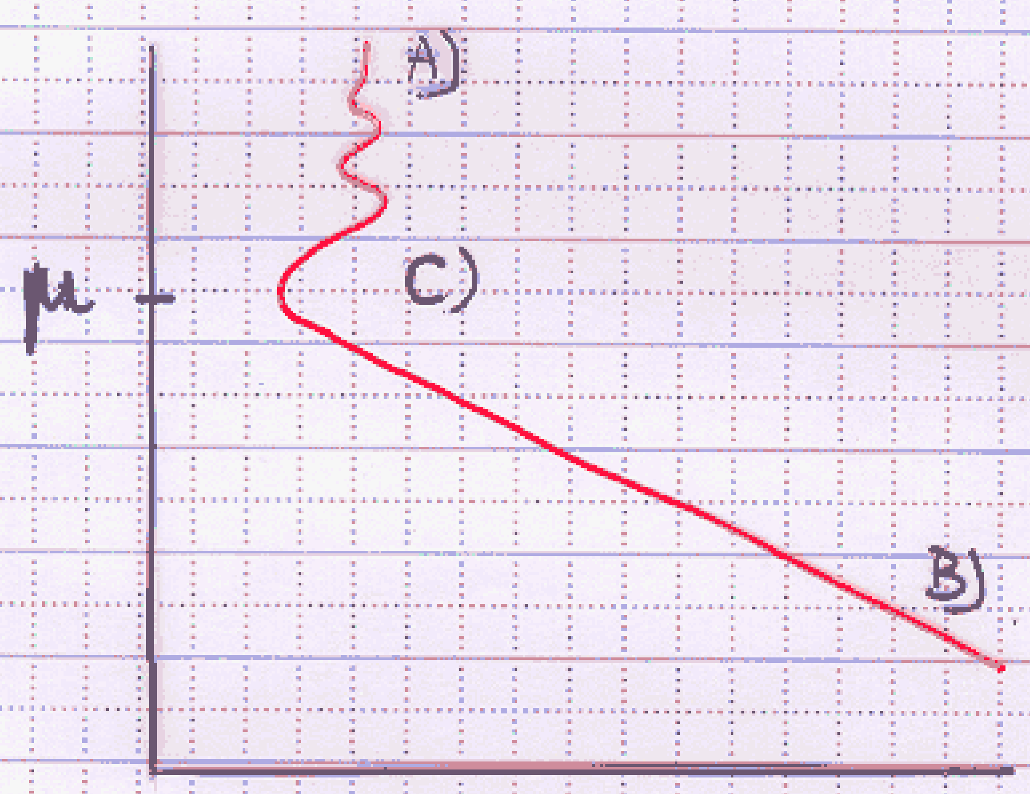
$\mu \gg \Delta m^2_{21} \qquad \mu \gtrsim \Delta m^2_{32}$

Matter effects $\Delta m_{21}^2 \ll \mu, \Delta m_{32}^2$

$$P^{matter}(\nu_e \to \nu_\mu) = s_{23}^2 \sin 2\theta_{13} \left(\frac{\Delta m_{32}^2}{2E_\nu B} \right)^2 \sin^2\left(\frac{LB}{2}\right)$$

$$B \equiv \sqrt{\left(\frac{\Delta m_{32}^2}{2E_\nu}\right)^2 + A^2 - 2A \frac{\Delta m_{32}^2}{2E_\nu} \cos 2\theta_{13}}$$

vacuum: $B \longleftrightarrow \frac{\Delta m_{32}^2}{2E_\nu}$



$L = 3000\,\mathrm{km}$

A) $\mu \ll \Delta m_{32}^2$ matter effects unimportant

B) $\Delta m_{32}^2 \lesssim \mu$, $\frac{\mu L}{2E_\nu} \ll 1$ matter effects unimportant

C) $\Delta m_{32}^2 \lesssim \mu$, $\frac{\mu L}{2E_\nu} \sim 1$ matter effects important

Sensitivities vs. E_μ, L $\left(\# cc \sim E_\mu^3 / L^2\right)$

Appearance

	$L < 3000\,km$	$L > 3000\,km$
Δm^2	$E_\mu^{-1/2}$	$E_\mu^{-1/2}\left(\frac{AL/2}{\sin\frac{AL}{2}}\right)^2$
Angles	$L\,E_\mu^{-3/2}$	$L\,E_\mu^{-3/2}$

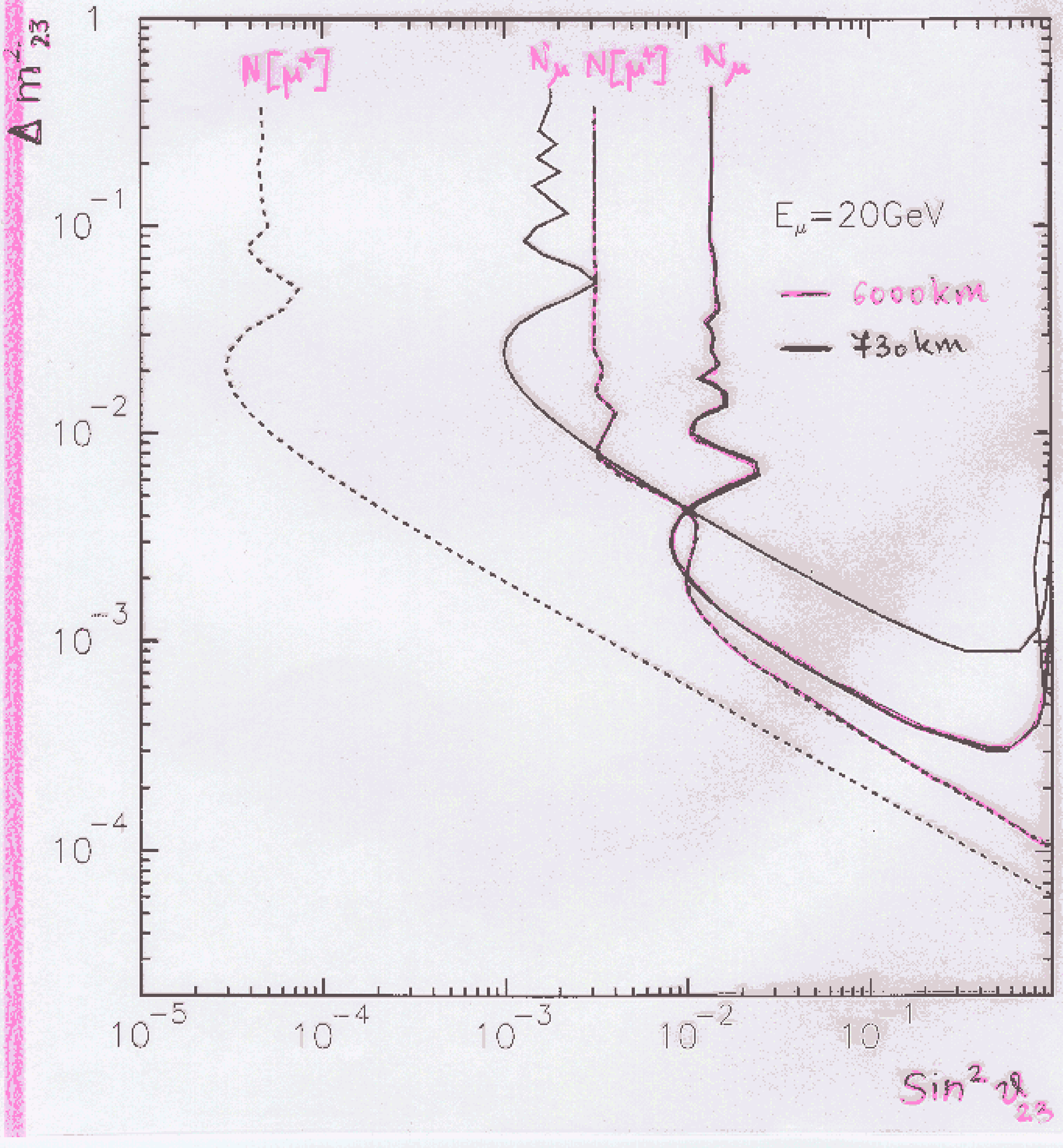
Disappearance

	$L < 3000\,km$	$L > 3000\,km$
Δm^2	$E_\mu^{1/4}\,L^{-1/2}$	More complicated due to competition between various terms.
Angles	$L^{1/2}\,E_\mu^{-3/4}$	

Three-family mixing

10 kTon

only statistical errors!



CP VIOLATION

How large?

Where to look?

Majorana phases unobservable $\propto \frac{m_{\nu_i}}{E}$

Dirac phase

$$A_{\not{CP}} = \frac{P(\nu_i \to \nu_j) - P(\bar{\nu}_i \to \bar{\nu}_j)}{+}$$

$$A_T = \frac{P(\nu_i \to \nu_j) - P(\nu_j \to \nu_i)}{+}$$

Equivalent in vacuum by CPT

$A_{\not{CP}} = A_T = 0$ if $\Delta m^2_{ij} = 0$ or $s_{ij} = 0$

$$P(\nu_\alpha \to \nu_\beta) = -4 \sum_{k>j} \mathrm{Re}\left[V_{\alpha j} V^*_{\beta j} V^*_{\alpha k} V_{\beta k}\right] \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)$$

$\bar{\nu}_\alpha \to \bar{\nu}_\beta$

$$\pm \frac{1}{2} \sum_{k>j} \mathrm{Im}\left[V_{\alpha j} V^*_{\beta j} V^*_{\alpha k} V_{\beta k}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)$$

$\simeq$

$$\frac{1}{8} c_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \sin\delta \; \Delta m^2_{12} \, \Delta m^2_{13} \, \Delta m^2_{23} \left(\frac{L}{2E}\right)^3$$

Mass suppression (in vacuum)

Numerator $\quad J \quad 2\sin^2 \frac{\Delta m_{32}^2 L}{4E} \quad \frac{\Delta m_{21}^2 L}{2E}$

Denominator $\quad$ Angles $\sin^2 \frac{\Delta m_{32}^2 L}{4E}$

$$A^{CP} \propto \frac{\Delta m_{21}^2 L}{2E}$$

$$\tilde{A}_{C\!\!\!/P\!\!\!/} = \frac{N[\mu^-]/N_o[e^-]\Big)_+ - N[\mu^+]/N_o[e^+]\Big)_-}{N[\mu^-]/N_o[e^-]\Big)_+ + N[\mu^+]/N_o[e^+]\Big)_-}$$

$\overset{?}{P}(\nu_e \to \nu_\mu) \qquad \overset{?}{P}(\bar{\nu}_e \to \bar{\nu}_\mu)$

$$\frac{\tilde{A}^{C\!\!\!/P\!\!\!/}}{\delta \tilde{A}^{C\!\!\!/P\!\!\!/}_{stat}} \propto E^{1/2} \left| \sin \frac{\Delta m_{32}^2 L}{4E} \right| \Delta m_{21}^2$$

Maximal $C\!\!\!/P\!\!\!/$ at $\frac{\Delta m_{32}^2 L}{4E} = \frac{\pi}{2}$

GeV with maximal CP violation, $\delta = 90^0$, and with various parameter values chosen in their currently allowed domains. The asymmetries in the Table are $A^{CP}_{e\mu}$[vac] calculated in vacuum, the "observed" asymmetry $A^{CP}_{e\mu}(\delta)$ of Eq.(19) (that includes matter and genuine CP-odd effects effects), the "theoretical" expectation $A^{CP}_{e\mu}(0)$ for the apparent CP-odd asymmetry induced by matter, and the genuine CP-odd asymmetry in matter:

$$\mathcal{A}_{e\mu} = A^{CP}_{e\mu}(\delta) - A^{CP}_{e\mu}(0)\,, \tag{21}$$

from which the matter effect is subtracted.

CP-odd asymmetries at $L = 732$ km, $\delta = \pi/2$.					
$\sin^2\theta_{12}$	θ_{13}	Δm^2_{12}	$A^{CP}_{e\mu}$[vac]	$\mathcal{A}_{e\mu}$	$A^{CP}_{e\mu}(0)$
0.5	13^0	10^{-5}	$-5.9\,10^{-3}$	$-5.5\,10^{-3}$	$1.6\,10^{-2}$
$5\,10^{-3}$	30^0	10^{-4}	$-3.4\,10^{-3}$	$-3.2\,10^{-3}$	$9.8\,10^{-3}$
0.5	30^0	10^{-4}	$-2.6\,10^{-2}$	$-2.5\,10^{-2}$	$7.8\,10^{-3}$
0.5	13^0	10^{-4}	$-5.6\,10^{-2}$	$-5.4\,10^{-2}$	$1.4\,10^{-2}$

Table 1: The CP asymmetries defined in the text, for $\theta_{23} = 45^0$, $\Delta m^2_{23} = 10^{-3}$ eV^2, $E_\nu = 7$ GeV and choices of other parameters compatible with solar and atmospheric data.

With no further ado, Table 1 conveys the message that, if Δm^2_{21} is indeed as small as the ensemble of solar neutrino experiments would imply, the CP-odd effects are only sizable in a small domain of parameter space, exemplified here by the last two rows of the table. Is that region amenable to empiric scrutiny?

A first question concerns the relative size of the measured and the theoretically subtracted terms. For the subtraction procedure to be useful θ_{23}, θ_{13}, Δm^2_{23} and the density profile traversed by the beam must be known with sufficient precission for the error in the subtracted term not to dominate the result. At the distance of $L = 732$ km used to construct Table 1, that does not seem to be a problem: for the parametres values of the last two rows, the subtractions are small enough that a precission of a factor of two in their detemination would suffice.

In matter

$\tilde{A}_{\not{CP}} \neq 0$ even if $\delta = 0$

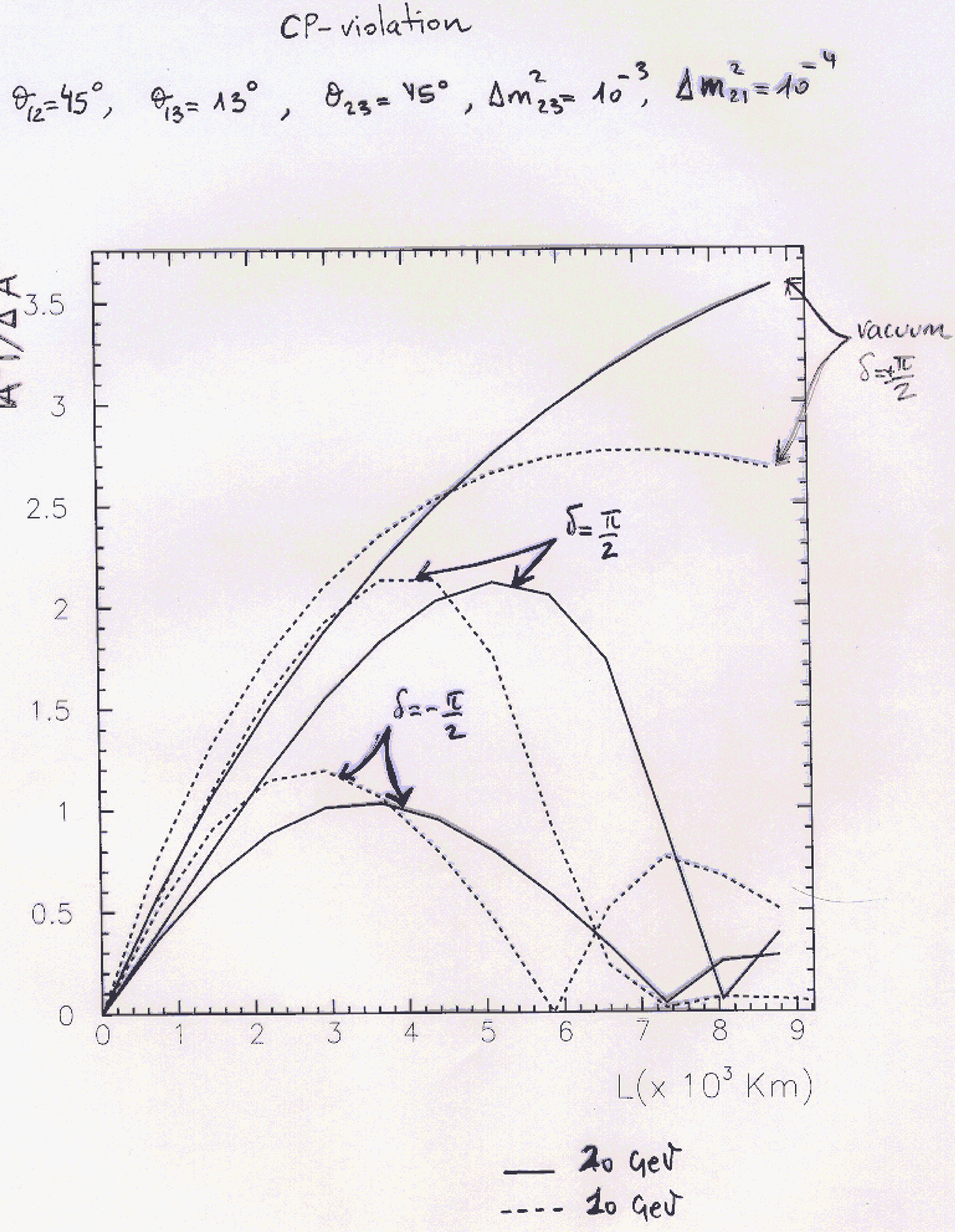
$P(\nu_\alpha \to \nu_\beta) = F(\mu) \qquad P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = F(-\mu)$

subtracted asymmetry

$\tilde{A}_{\not{CP}}(\pi/2) - \tilde{A}_{\not{CP}}(0) \propto \sin\delta$

To leading order in Δm^2_{21} and $\Delta m^2_{32} < |\mu|$

$$\frac{\tilde{A}_{\not{CP}}(\pi/2) - \tilde{A}_{\not{CP}}(0)}{\delta \tilde{A}_{\not{CP}}} \propto E^{1/2} \frac{\Delta m^2_{32}}{|\mu|} \Delta m^2_{21} \, F\left(\frac{|\mu| L}{4E}\right)$$



CP-violation
$\theta_{12}=45°$, $\theta_{13}=13°$, $\theta_{23}=45°$, $\Delta m^2_{23}=10^{-3}$, $\Delta m^2_{21}=10^{-4}$
$|\tilde{A}^{CP}|/\Delta\tilde{A}^{CP}$
vacuum
$\delta=\pm\frac{\pi}{2}$
$\delta=\frac{\pi}{2}$
$\delta=-\frac{\pi}{2}$
L(x 10^3 Km)
20 GeV
10 GeV

CONCLUSIONS

If wrong-sign μ background under control, ν factory from μ-accumulator

① Severely test SK

② Measure with good accuracy (or severely constrain)

Δm^2_{23}

θ_{23}

θ_{13}

③ Extracting information on θ_{12}, Δm^2_{12}, δ seems statistically impossible if Δm^2_{12} is compatible with all solar data.