CP Violation at $\mu^+\mu^-$ Colliders

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- Discrete symmetries: P, C, T, CP, CPT
- Higgs scalar–pseudoscalar transitions and resonant CP violation at muon colliders:
 - Mechanism
 - Theoretical models
- Radiative Higgs-sector CP violation in the MSSM
 - CP-violating self-energy effects
 - CP-violating vertex effects
- CP asymmetries at $\mu^+\mu^-$ colliders
- Conclusions Outlook

• Discrete symmetries: P, C, T, CP, CPT

Discrete transformations applied to a state $S^0(\mathbf{p})$, e.g. $S^0 = \{K^0, B^0, \ldots\}$

In general, invariance under CP requires [H, CP] = 0, which implies:

$$\langle f(\mathbf{p}_f, \mathbf{s}_f) | e^{-iHt} | i(\mathbf{p}_i, \mathbf{s}_i) \rangle = \langle \bar{f}(-\mathbf{p}_{\bar{f}}, \mathbf{s}_{\bar{f}}) | e^{-iHt} | \bar{i}(-\mathbf{p}_{\bar{i}}, \mathbf{s}_{\bar{i}}) \rangle$$

Invariance under CPT requires $\left[\text{CPT } H (\text{CPT})^{-1} \right]^{\dagger} = H$, implying that

$$\langle f(\mathbf{p}_f, \mathbf{s}_f) | e^{-iHt} | i(\mathbf{p}_i, \mathbf{s}_i) \rangle = \langle \overline{i}(\mathbf{p}_{\overline{i}}, -\mathbf{s}_{\overline{i}}) | e^{-iHt} | \overline{f}(\mathbf{p}_{\overline{f}}, -\mathbf{s}_{\overline{f}}) \rangle^*$$

• Higgs scalar-pseudoscalar transitions and resonant CP violation at muon colliders

– Mechanism



[J. Papavassiliou and A.P., PRL**75** (1995) 3060; PR**D53** (1996) 2128; PR**D54** (1996) 5315; PRL**80** (1998) 2785; PR**D58** (1998) 053002.]

- $V_i^{P,D} = |V_i^{P,D}| e^{i\delta_f} e^{i\phi_w} \xrightarrow{\text{CP}} \bar{V}_i^{P,D} = |\bar{V}_i^{P,D}| e^{i\delta_f} e^{-i\phi_w}$ (ε' -type effects)
- $\mathcal{H}(s) \xrightarrow{\mathrm{CP}} \overline{\mathcal{H}}(s) = \mathcal{H}^T(s)$ in a (K^0, \overline{K}^0) -like basis (ε -type effects)

Observables of CP asymmetry:

$$\mathcal{A}_{\mathbf{CP}} = \frac{|\mathcal{T}|^2 - |\mathcal{T}^{\mathbf{CP}}|^2}{|\mathcal{T}|^2 + |\mathcal{T}^{\mathbf{CP}}|^2} \approx \frac{|\mathcal{T}^{\mathbf{res}}|^2 - |\overline{\mathcal{T}}^{\mathbf{res}}|^2}{|\mathcal{T}^{\mathbf{res}}|^2 + |\overline{\mathcal{T}}^{\mathbf{res}}|^2}$$

Higgs scalar – pseudoscalar mixing

The inverse resummed propagator HA matrix reads:

$$s\mathbf{1} - \mathcal{H}(s) = s\mathbf{1} - \begin{pmatrix} M_A^2 - \widehat{\Pi}_{AA} & -\widehat{\Pi}_{AH} \\ -\widehat{\Pi}_{HA} & M_H^2 - \widehat{\Pi}_{HH} \end{pmatrix}$$

where $H = \frac{1}{\sqrt{2}} \left(S^0 + \bar{S}^0 \right), \ iA = \frac{1}{\sqrt{2}} \left(S^0 - \bar{S}^0 \right),$ and $S^0 \xrightarrow{\text{CP}} \bar{S}^0.$

In the weak (S^0,\bar{S}^0) basis, the respective inverse resummed propagator matrix $\widetilde{H}(s)$ is

$$\frac{1}{2} \begin{pmatrix} M_{H}^{2} + M_{A}^{2} - \widehat{\Pi}_{HH} - \widehat{\Pi}_{AA} & M_{H}^{2} - M_{A}^{2} - \widehat{\Pi}_{HH} + \widehat{\Pi}_{AA} + 2i\widehat{\Pi}_{HA} \\ M_{H}^{2} - M_{A}^{2} - \widehat{\Pi}_{HH} + \widehat{\Pi}_{AA} - 2i\widehat{\Pi}_{HA} & M_{H}^{2} + M_{A}^{2} - \widehat{\Pi}_{HH} - \widehat{\Pi}_{AA} \end{pmatrix}$$

 $\begin{array}{ll} \mathsf{CPT} \text{ invariance } \Rightarrow \widetilde{H}_{11} = \widetilde{H}_{22} \\ \mathsf{CP} \text{ invariance } \Rightarrow \widetilde{H}_{12} = \widetilde{H}_{21} \end{array}$

Indirect CP violation:

$$\begin{aligned} \left| \frac{q}{p} \right|^2 &= \left| \frac{\tilde{H}_{21}}{\tilde{H}_{12}} \right| \\ &= \left[\frac{(M_H^2 - M_A^2 - 2\mathrm{Im}\,\widehat{\Pi}_{HA})^2 + (\mathrm{Im}\,(\widehat{\Pi}_{HH} - \widehat{\Pi}_{AA}) + 2\mathrm{Re}\,\widehat{\Pi}_{HA})^2}{(M_H^2 - M_A^2 + 2\mathrm{Im}\,\widehat{\Pi}_{HA})^2 + (\mathrm{Im}\,(\widehat{\Pi}_{HH} - \widehat{\Pi}_{AA}) - 2\mathrm{Re}\,\widehat{\Pi}_{HA})^2} \right]^{1/2} \end{aligned}$$

Conditions for resonant CP violation through mixing [A.P., NP**B504** (1997) 61.]

•
$$\operatorname{Re}\widehat{\Pi}_{HA} \neq 0$$
, $\operatorname{Im}\widehat{\Pi}_{HA} = 0$

$$\frac{q}{p}\Big|^{2} \xrightarrow{M_{H} \sim M_{A}} \frac{\operatorname{Im}\left(\widehat{\Pi}_{HH} - \widehat{\Pi}_{AA}\right) + 2\operatorname{Re}\widehat{\Pi}_{HA}}{\operatorname{Im}\left(\widehat{\Pi}_{HH} - \widehat{\Pi}_{AA}\right) - 2\operatorname{Re}\widehat{\Pi}_{HA}}$$
$$\xrightarrow{M_{H} \gg M_{A}} 1$$

Resonant CP violation for $\operatorname{Im}(\widehat{\Pi}_{HH} - \widehat{\Pi}_{AA}) \sim 2\operatorname{Re}\widehat{\Pi}_{HA}$.

• Im
$$\widehat{\Pi}_{H\mathbf{A}} \neq 0$$
, Re $\widehat{\Pi}_{H\mathbf{A}} = 0$

$$\left| \frac{q}{p} \right|^2 \xrightarrow{M_H \sim M_A} \frac{M_H^2 - M_A^2 - 2 \operatorname{Im} \widehat{\Pi}_{HA}}{M_H^2 - M_A^2 + 2 \operatorname{Im} \widehat{\Pi}_{HA}}$$
$$\xrightarrow{M_H \gg M_A} 1$$

Resonant CP violation for $M_H^2 - M_A^2 \sim 2 \text{Im} \widehat{\Pi}_{HA}$.

The general condition for resonant CP violation is given by

$$|M_H^2 - M_A^2 - \widehat{\Pi}_{HH} + \widehat{\Pi}_{AA}| \lesssim 2 |\widehat{\Pi}_{AH}|$$

- Theoretical models: How to get a large HA mixing?

 Explicit or spontaneous CP violation in the Higgs potential at the tree level, e.g. 2HDM. [T.D. Lee, PRD8 (1973) 1226; S. Weinberg, PRL37 (1976) 657; G.C. Branco, PRL44 (1980) 504, ...]



 $H \xrightarrow{A}$ The CP-violating HA mixing occurs at the tree level, but generically M / Mtree level, but generically $M_H \not\sim M_A$.

- Spontaneous or explicit radiative CP violation in the Higgs potential.
 - Spontaneous radiative CP violation: it generically leads to a very light 'CP-odd' scalar, with $M_A \lesssim 40$ GeV, and is phenomenologically highly disfavoured. [H. Georgi, G. Pais, PRD10 (1974) 1246; J.C. Romao, PLB173 (1986) 309]
 - Explicit radiative CP violation:
 - (i) Through loop effects of heavy Majorana neutrinos in a constrained 2HDM potential [A.P., PRL77 (1996) 4996.]



 $\begin{array}{c} H \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array} \\ \begin{array}{c} A \\ \hline \end{array} \\ \hline \end{array} \\ \begin{array}{c} \text{Natural CP-violating scenarios,} \\ \text{with } M_H - M_A \sim \Gamma_{H,A}. \end{array}$

(ii) Through radiative effects of stops/sbottoms in the MSSM. [A.P., PRD58 (1998) 096010; PLB435 (1998) 88]

• Radiative Higgs-sector CP violation in the MSSM

Two major effects of CP violation on the Higgs sector:

- CP-violating self-energy effects
- CP-violating vertex effects
- CP-violating self-energy effects:



$$\mathcal{M}_{SP}^2 \sim \frac{m_t^4}{v^2} \frac{\operatorname{Im}(\mu A_t)}{32\pi^2 Q_t^2}$$
$$\times \left(1, \frac{|A_t|^2}{Q_t^2}, \frac{|\mu|^2}{\tan\beta Q_t^2}, \frac{2\operatorname{Re}(\mu A_t)}{Q_t^2}\right)$$
$$\lesssim (100 \text{ GeV})^2$$

[A.P., PR**D58** (1998) 096010; PL**B435** (1998) 88;

A.P., C.E.M. Wagner, NPB553 (1999) 3;

M. Carena, J. Ellis, A.P., C.E.M. Wagner, NP**B586** (2000) 92; hep-ph/0009212; D.A. Demir, PR**D60** (1999) 055006; S.Y. Choi, M. Drees, J.S. Lee, PL**B481** (2000) 57; G.L. Kane and L.-T. Wang, hep-ph/0003198; T. Ibrahim and P. Nath, hep-ph/0008237] :

The mixing of the three neutral Higgs bosons

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ a \end{pmatrix} = O \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}$$

O is a 3×3 orthogonal matrix which <u>also</u> describes the mixing of the Higgs bosons with different CP parities.

In analogy to the case of neutrinos and quarks, Higgs bosons with mixed CP parities are ordered according to their weights:

$$M_{H_1}~\leq~M_{H_2}~\leq~M_{H_3}$$

At the one-loop level, M_{H_i} (with i = 1, 2, 3) and O are analytically determined by the input parameters:

$$egin{aligned} &M_{H^+}(m_t)\,,\quad aneta(m_t)\,,\ &\mu(m{Q}_{tb})\,,\quad A_t(m{Q}_{tb})\,,\quad A_b(m{Q}_{tb})\,,\ &\widetilde{M}_Q^2(m{Q}_{tb})\,,\quad \widetilde{M}_t^2(m{Q}_{tb})\,,\quad \widetilde{M}_b^2(m{Q}_{tb})\,. \end{aligned}$$

CP-conserving versus CP-violating MSSM parameters:

CP-conserving:

aneta \widetilde{M}_Q^2 , \widetilde{M}_t^2 , \widetilde{M}_b^2 μ $A_{t,b}$ M_A

CP-violating:

 $egin{aligned} & aneta \ , \ {
m with} \ m\xi &= 0 \ & ilde{M}_Q^2 \ , \ & ilde{M}_t^2 \ , \ & ilde{M}_b^2 \ &|\mu| \ , \ & ext{ arg} \ (\mu) \ &|A_{t,b}| \ , \ & ext{ arg} \ (A_{t,b}) \ & ilde{M}_{H^+} \end{aligned}$

At two loops:

 $m_{ ilde{m{g}}}$

 $|m_{\tilde{g}}|$ and $rg\left(m_{\tilde{g}}
ight)$

- CP-violating vertex effects:

Effective H_ibb -coupling



 $-\mathcal{L}_{\phi^0\bar{b}b}^{\text{eff}} = (h_b + \delta h_b) \phi_1^{0*} \bar{b}_R b_L + \Delta h_b \phi_2^{0*} \bar{b}_R b_L + \text{h.c.}$ with

$$\frac{\delta h_b}{h_b} \sim -\frac{2\alpha_s}{3\pi} \frac{m_{\tilde{g}}^* A_b}{\max\left(Q_b^2, |m_{\tilde{g}}|^2\right)} - \frac{|h_t|^2}{16\pi^2} \frac{|\mu|^2}{\max\left(Q_t^2, |\mu|^2\right)}$$
$$\frac{\Delta h_b}{h_b} \sim \frac{2\alpha_s}{3\pi} \frac{m_{\tilde{g}}^* \mu^*}{\max\left(Q_b^2, |m_{\tilde{g}}|^2\right)} + \frac{|h_t|^2}{16\pi^2} \frac{A_t^* \mu^*}{\max\left(Q_t^2, |\mu|^2\right)}$$

and

$$h_b = \frac{g_w m_b}{\sqrt{2} M_W \cos\beta \left[1 + \frac{\delta h_b}{h_b} + \left(\frac{\Delta h_b}{h_b}\right) \tan\beta \right]}$$

[CP-conserving studies: R. Hempfling, PRD49 (1994) 6168;
 L. Hall, R. Rattazzi, U. Sarid, PRD50 (1994) 7048;
 M. Carena, M. Olechowski, S. Pokorski, C.E.M. Wagner, NPB426 (1994) 269;
 D. Pierce, J. Bagger, K. Matchev, R. Zhang, NPB491 (1997) 3.]

Effective H_itt -coupling



$$-\mathcal{L}_{\phi^0 \bar{t}t}^{\text{eff}} = \Delta h_t \,\phi_1^0 \,\bar{t}_R t_L + (h_t + \delta h_t) \,\phi_2^0 \,\bar{t}_R t_L + \text{h.c.}$$

with

$$\frac{\Delta h_t}{h_t} \sim \frac{2\alpha_s}{3\pi} \frac{m_{\tilde{g}}^* \mu^*}{\max\left(Q_t^2, |m_{\tilde{g}}|^2\right)} + \frac{|h_b|^2}{16\pi^2} \frac{A_b^* \mu^*}{\max\left(Q_b^2, |\mu|^2\right)}$$
$$\frac{\delta h_t}{h_t} \sim -\frac{2\alpha_s}{3\pi} \frac{m_{\tilde{g}}^* A_t}{\max\left(Q_t^2, |m_{\tilde{g}}|^2\right)} - \frac{|h_b|^2}{16\pi^2} \frac{|\mu|^2}{\max\left(Q_b^2, |\mu|^2\right)}$$

and

$$h_t = \frac{g_w m_t}{\sqrt{2}M_W \sin\beta \left[1 + \frac{\delta h_t}{h_t} + \left(\frac{\Delta h_t}{h_t}\right) \cot\beta\right]}$$

[CP-conserving studies: S. Heinemeyer, W. Hollik, G. Weiglein, PL**B440** (1998) 96; PR**D58** (1998) 091701; M. Carena, H.E. Haber, S. Heinmeyer, W. Hollik, C.E.M. Wagner, G. Weiglein, NP**B580** (2000) 29.]

Effective Higgs-boson couplings to gauge bosons and fermions



• CP asymmetries at $\mu^+\mu^-$ colliders





In a general Lorentz frame, the spin vector is given by

$$s^{\mu} = \left(\frac{\mathbf{sp}}{m}, \mathbf{s} + \frac{\mathbf{sp}}{(E+m)m}\right)$$

- CP violation in the transverse muon polarization:

[B. Grzadkowski, J.F. Gunion, Phys. Lett. **B350** (1995) 218; D. Atwood and A. Soni, Phys. Rev. **D52** (1995) 6271.]



CP-violating observable:

$$\mathcal{A}^{t}_{\mathbf{CP}} = \frac{\sigma(\mu^{-}(\vec{s}_{x})\mu^{+}(\vec{s}_{y}) \to f\bar{f}) - \sigma(\mu^{-}(\vec{s}_{x})\mu^{+}(-\vec{s}_{y}) \to f\bar{f})}{\sigma(\mu^{-}(\vec{s}_{x})\mu^{+}(\vec{s}_{y}) \to f\bar{f}) + \sigma(\mu^{-}(\vec{s}_{x})\mu^{+}(-\vec{s}_{y}) \to f\bar{f})}$$

Properties of \mathcal{A}^t_{CP} :

$$\begin{array}{rcl} \mathrm{CP}: \ \mathcal{A}^{t}{}_{\mathrm{CP}} & \rightarrow & -\mathcal{A}^{t}{}_{\mathrm{CP}} \\ \\ \mathsf{CP}'\mathsf{T}': \ \mathcal{A}^{t}{}_{\mathrm{CP}} & \rightarrow & \mathcal{A}^{t}{}_{\mathrm{CP}} \end{array}$$

 \mathcal{A}^t_{CP} is a CP'T'-even observable and to leading order, is induced by dispersive parts.

In general, for $\mathcal{L}_{\mathrm{int}} = H \; ar{\mu} \Big(g^S \; + \; i g^P \gamma_5 \Big) \mu$, we have

$$\begin{split} \bar{\sigma}(\zeta) &= \bar{\sigma}^{(\text{unpol})} \left[1 + P_L^+ P_L^- + P_T^+ P_T^- \right. \\ &\times \left(\frac{(g^S)^2 - (g^P)^2}{(g^S)^2 + (g^P)^2} \cos \zeta - \frac{2(g^S)(g^P)}{(g^S)^2 + (g^P)^2} \sin \zeta \right) \right], \end{split}$$

where $\zeta = \phi_+ - \phi_-$ is the relative angle of the transverse polarizations. So, $\mathcal{A}^t{}_{\rm CP}$ is given by

$$\mathcal{A}^{t}_{CP} = \frac{\sigma(\zeta = \frac{\pi}{2}) - \sigma(\zeta = -\frac{\pi}{2})}{\sigma(\zeta = \frac{\pi}{2}) + \sigma(\zeta = -\frac{\pi}{2})} = -P_{T}^{+}P_{T}^{-}\frac{2(g^{S})(g^{P})}{(g^{S})^{2} + (g^{P})^{2}}$$

- CP violation in the longidutinal muon polarization:

[A.P., Phys. Rev. Lett. 77 (1996) 4996; Nucl. Phys. B504 (1997) 61]



$$\mathcal{A}^{l}_{\mathbf{CP}} = \frac{\sigma(\mu_{\mathrm{RH}}^{-}\mu_{\mathrm{RH}}^{+} \to f\bar{f}) - \sigma(\mu_{\mathrm{LH}}^{-}\mu_{\mathrm{LH}}^{+} \to f\bar{f})}{\sigma(\mu_{\mathrm{RH}}^{-}\mu_{\mathrm{RH}}^{+} \to f\bar{f}) + \sigma(\mu_{\mathrm{LH}}^{-}\mu_{\mathrm{LH}}^{+} \to f\bar{f})}$$
$$\frac{\mathrm{CP}: \mathcal{A}^{l}_{\mathbf{CP}} \to -\mathcal{A}^{l}_{\mathbf{CP}}$$

Naive 'T'-reversal transformation:



 \mathcal{A}^{l}_{CP} is a CP'T'-odd observable and to leading order, is induced by absorptive parts.



$$\mathcal{A}^{l}_{CP} \sim -\frac{2\widehat{\Pi}^{AH} (\operatorname{Im} \widehat{\Pi}^{HH} - \operatorname{Im} \widehat{\Pi}^{AA})}{(M_{H}^{2} - M_{A}^{2})^{2} + (\operatorname{Im} \widehat{\Pi}^{HH})^{2} + (\operatorname{Im} \widehat{\Pi}^{AA})^{2}}$$

at $\sqrt{s} \approx M_H \approx M_A$.

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[E. Asakawa, S.Y. Choi, J.S. Lee, hep-ph/0005118.]



$$\begin{split} M_{H^+} &\approx 0.4 \; \mathrm{TeV}, \; \mathrm{tan} \; eta = 3, \; |\mathbf{A_t}| = |\mathbf{A_b}| = 1 \; \mathrm{TeV}, \\ |\mu| &= 1 \; \mathrm{TeV}, \; \widetilde{M}_Q^2 = \widetilde{M}_t^2 = \widetilde{M}_b^2 = 0.5 \; \mathrm{TeV}; \\ \mathrm{arg} \; (\mu \mathbf{A_{t,b}}) &= 0 \; \mathrm{(solid)}, \; 90^\circ \; \mathrm{(dashed)} \end{split}$$

[E. Asakawa, S.Y. Choi, J.S. Lee, hep-ph/0005118.]



$$\begin{split} M_{H^+} &\approx 0.4 \text{ TeV}, \ an eta = 10, \ |m{A_t}| = |m{A_b}| = 1 \text{ TeV}, \\ |\mu| &= 1 \text{ TeV}, \ \widetilde{M}_Q^2 = \widetilde{M}_t^2 = \widetilde{M}_b^2 = 0.5 \text{ TeV}; \\ &rg \left(\mu m{A_{t,b}}\right) = 0 \ \text{(solid)}, \ 90^\circ \ \text{(dashed)} \end{split}$$

- Conclusions Outlook
- Resonant CP violation at Higgs scalar-pseudoscalar transitions is the basic mechanism for enhanced CP asymmetries at muon colliders.
- An appealing theoretical framework for such studies is the MSSM with explicit radiative CP violation, where the heaviest 'CP-even' Higgs boson and the 'CP-odd' scalar can naturally be nearly degenerate of the order of their widths.
- Polarization of the μ^- and μ^+ beams is very valuable for determining the CP nature of a Higgs boson or for analyzing a two-Higgs-boson-mixing system. The current effective degree of polarization proposed is $P \lesssim 0.4$.

[B. Grzadkowski, J.F. Gunion, J. Pliszka, hep-ph/0003091.]

• Further studies are necessary on both theory and experiment sides, including realistic background analyses due to γ , Z-exchange graphs.