Pilar HERNANDEZ

From discussions with

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Three family mixing

Angle determination

CP violation

TWO-FAMILY MIXING $\nu_\mu \leftrightarrow \nu_\tau$

$5\cdot 10^{-4} \lesssim \Delta m^2_{23} \lesssim 5\cdot 10^{-3}$ $\qquad \sin^2 2\vartheta_{23} \sim 1$

CERN-GS $\qquad E_{\nu_\mu} \lesssim 5\,\mathrm{GeV}$ $\qquad \nu_\mu$ disappearance

(At larger distances ν_τ appearance experiments possible!)

ν_μ disappearance

LBL (Fermilab, GS) error dominated by systematics $\qquad$ Flux $\qquad$ Efficiencies $\qquad$ Near detector?

μ-booster $\qquad$ hep-ph/9808485 Bueno, Campanelli, Rubbia

Good knowledge of the fluxes $\Rightarrow$ error dominated by systematics

$$\text{Sensitivity} \sim \frac{1}{\sqrt{N}}$$

No advantage in charge identification

$\mu^- \to \nu_\mu \bar{\nu}_e e^-$

- $\bar{\nu}_e \to e^+$
- ν_τ
- $\nu_\mu \to \mu^-$

$$N_\mu = (1 - P_{\nu_\mu\nu_\tau})\, F_{\nu_\mu}\cdot \sigma_{\nu_\mu}$$

$$N_e = F_{\bar{\nu}_e}\cdot \sigma_{\bar{\nu}_e}$$

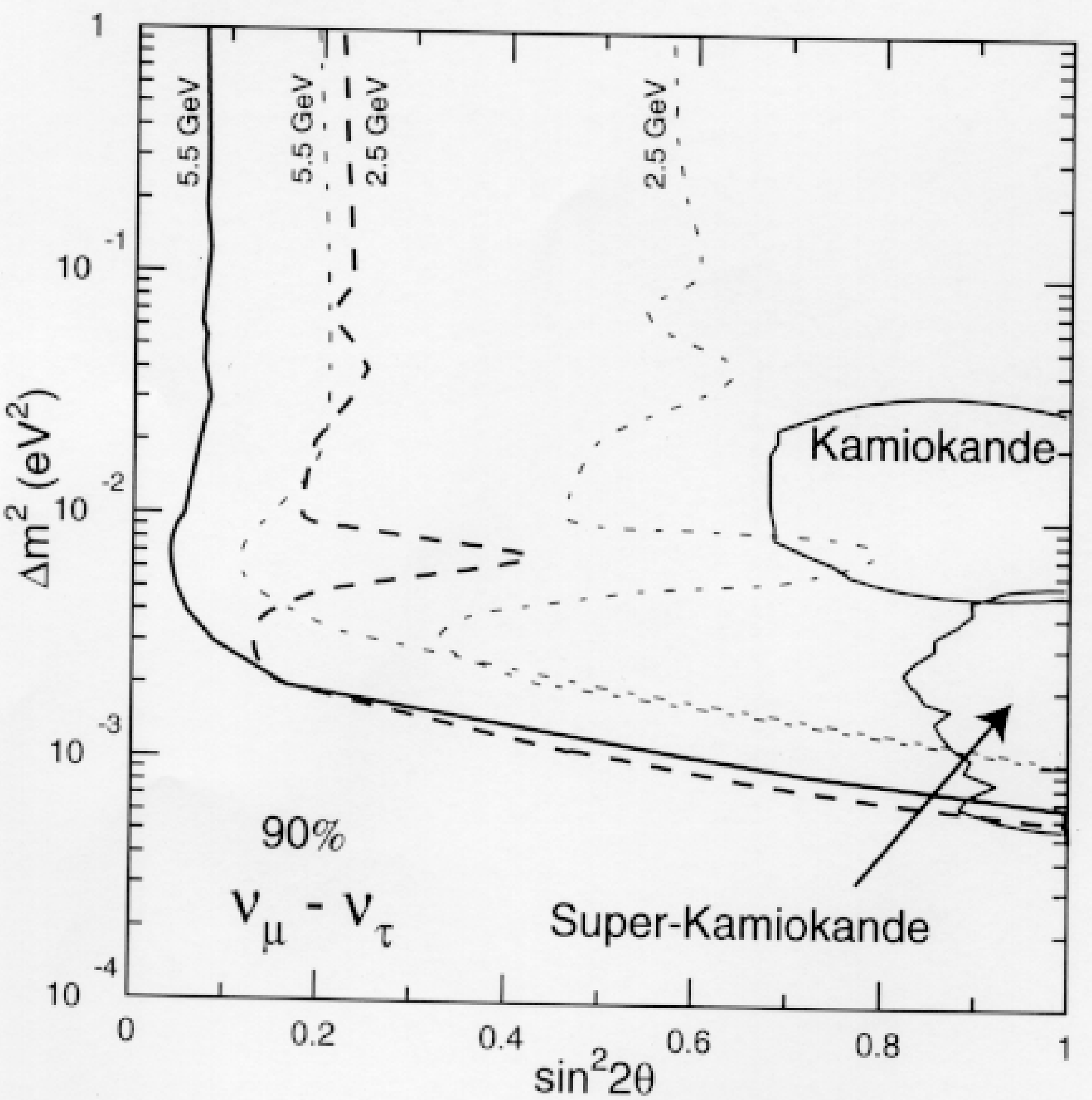


Figure 8: 90% C.L contours for a disappearance experiment with a total flux of 10^{21} and 10^{22} muons, compared to the SuperKamiokande 90% C.L. contour. Full line is the limit for 5.5 GeV muon beam and 10^{22} muons, dashed bold line is for 2.5 GeV beam and 10^{22} muons, dashed line is for 5.5 GeV beam and 10^{21} muons, dot-dashed line is for 2.5 GeV beam and 10^{21} muons.

THREE-FAMILY MIXING

	# angles	# phases
Dirac Fermions	$\frac{N(N-1)}{2} = 3$	$\frac{(N-2)(N-1)}{2} = 1$

$$\begin{pmatrix} V_{\nu_e e} & V_{\nu_e \mu} & V_{\nu_e \tau} \\ V_{\nu_\mu e} & V_{\nu_\mu \mu} & V_{\nu_\mu \tau} \\ V_{\nu_\tau e} & V_{\nu_\tau \mu} & V_{\nu_\tau \tau} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\delta} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\delta} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\delta} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\delta} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}$$

Majorana Fermions	$\frac{N(N-1)}{2} = 3$	$\frac{N(N-1)}{2} = 3$

$$\text{MM.} = \text{CKM} \begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}$$

Observables that depend on the Majorana phases are suppressed by factors

$$\frac{M_{\nu_i}}{E} \rightarrow \text{observation is quite hopeless!}$$

Minimal Scheme: Solar $\oplus$ Atmospheric

$$|\Delta m^2_{23}| \gg |\Delta m^2_{12}|$$

Atmospheric osc. | Solar osc.

Atmospheric osc.	Solar osc.
Δm^2_{23}	Δm^2_{12}
θ_{23}	θ_{12}
θ_{13}	θ_{13}

$$\rightarrow P_{\nu_e \nu_\mu} = 4\, s^2_{23}\, s^2_{13}\, c^2_{13} \sin^2 \frac{\Delta m^2_{23} L}{4E}$$

$$P_{\nu_e \nu_\tau} = 4\, c^2_{23}\, s^2_{13}\, c^2_{13} \sin^2 \frac{\Delta m^2_{23} L}{4E}$$

$$\rightarrow P_{\nu_\mu \nu_\tau} = 4\, c^4_{13}\, s^2_{23}\, c^2_{23} \sin^2 \frac{\Delta m^2_{23} L}{4E}$$

observables

μ- booster	$N_{e^+}, N_{e^-}, N_{\mu^+}, N_{\mu^-}$	Charge id.
	N_e, N_μ	No charge id.
LBL	N_μ, N_e?	

Sensitivity

$$\mu^- \to \nu_\mu \, \bar{\nu}_e \, e^-$$

- $\bar{\nu}_e$:
  - $\to \bar{\nu}_e \to e^+$ disappearance
  - $\to \bar{\nu}_\mu \to \mu^+$ appearance
  - $\to \bar{\nu}_\tau$
- ν_μ:
  - $\to \nu_e \to e^-$ appearance
  - $\to \nu_\mu \to \mu^-$ disappearance
  - $\to \nu_\tau$

disappearance sensitivity $\sim \frac{1}{\sqrt{N}}$

appearance sensitivity $\sim \frac{1}{N}$

Barbieri et al.

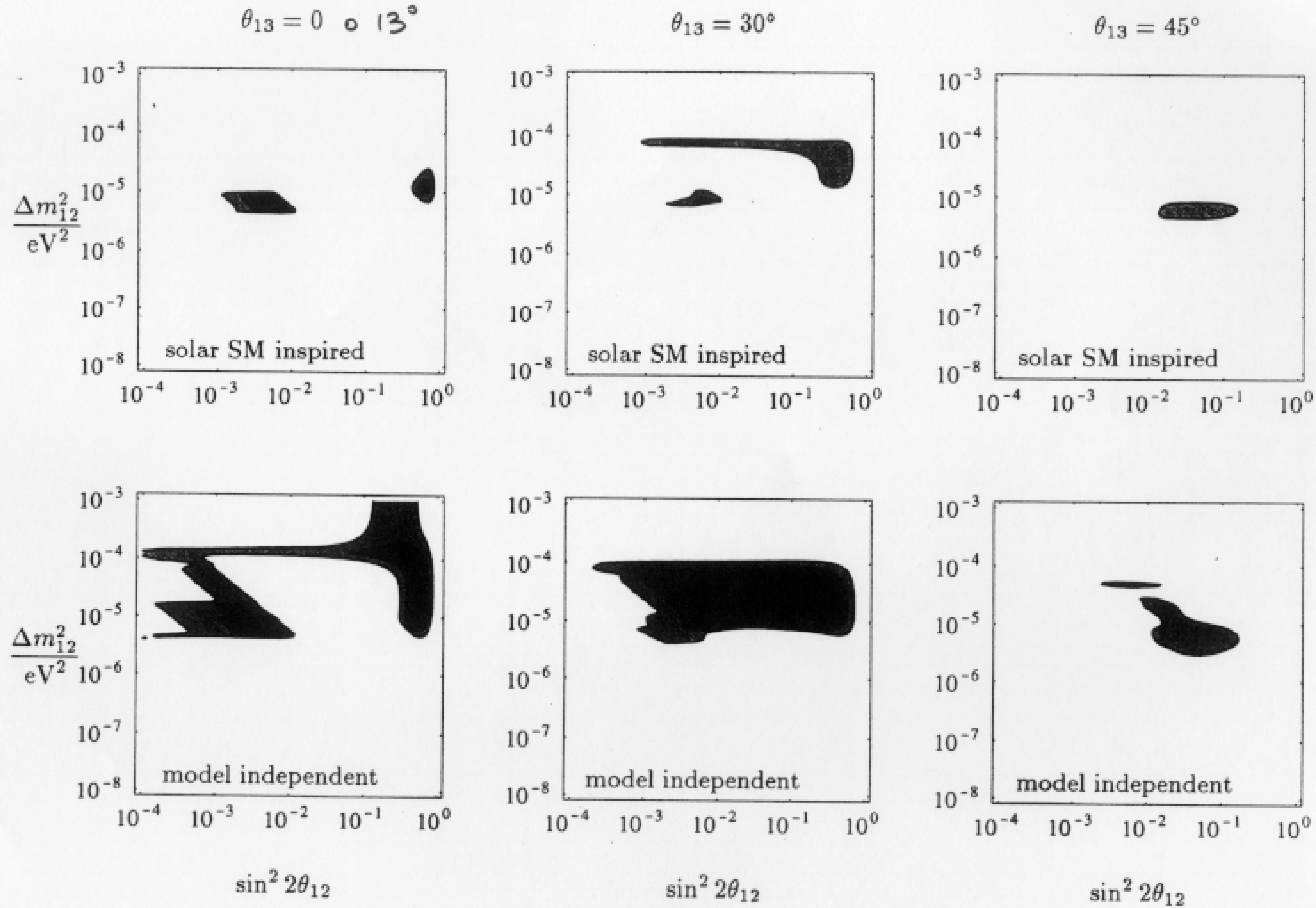
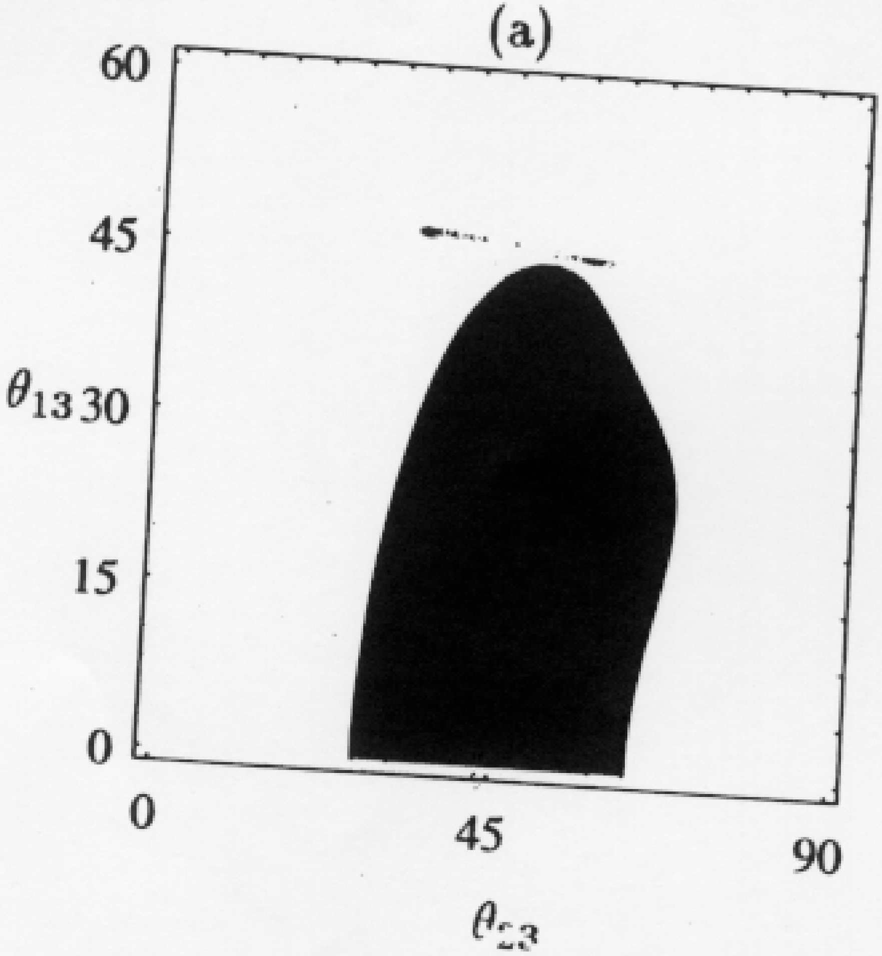


Figure 3: *Allowed regions in the plane* $(\sin^2 2\theta_{12}, \Delta m^2_{12})$ *for* $\theta_{13} = 0, 30°$ *and* $45°$. *The upper plots assume that*

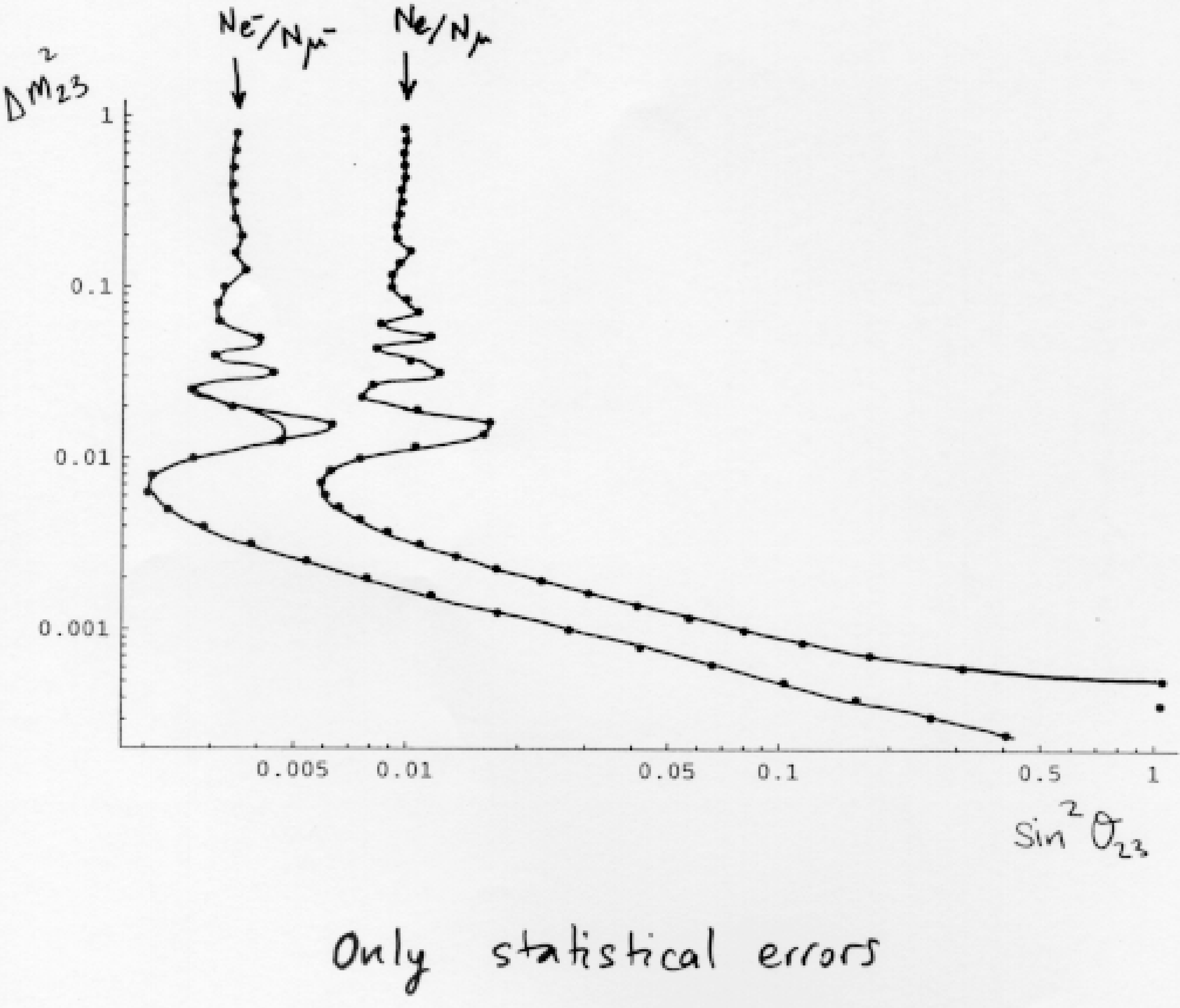


SK Limit $\Delta m^2_{12} \ll 10^{-3}$

any ϑ_{12}

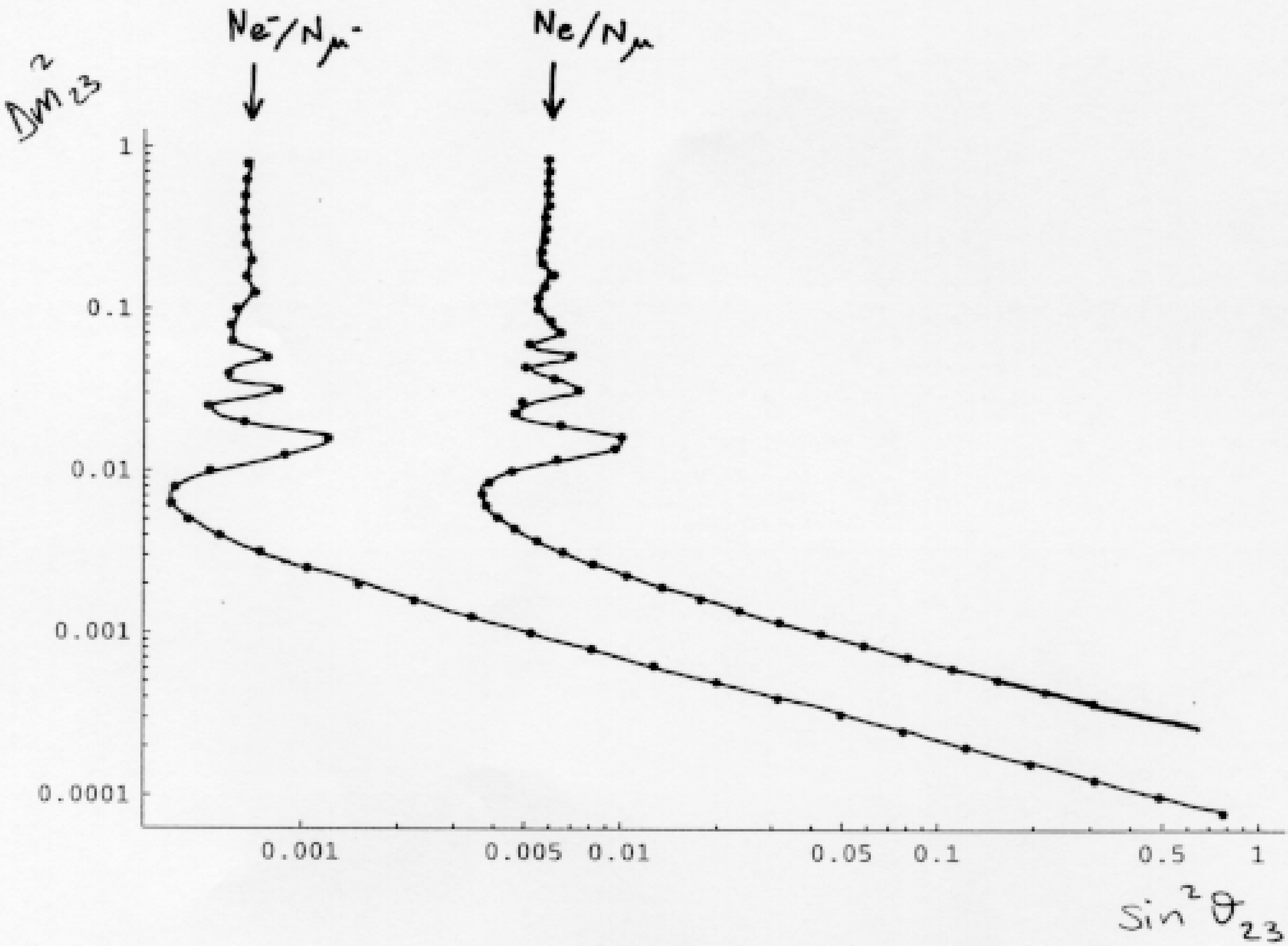
hep-ph/9807235 Barbieri et al.

$\theta_{13} = 13^\circ$



Only statistical errors

$\vartheta_{13} = 45°$



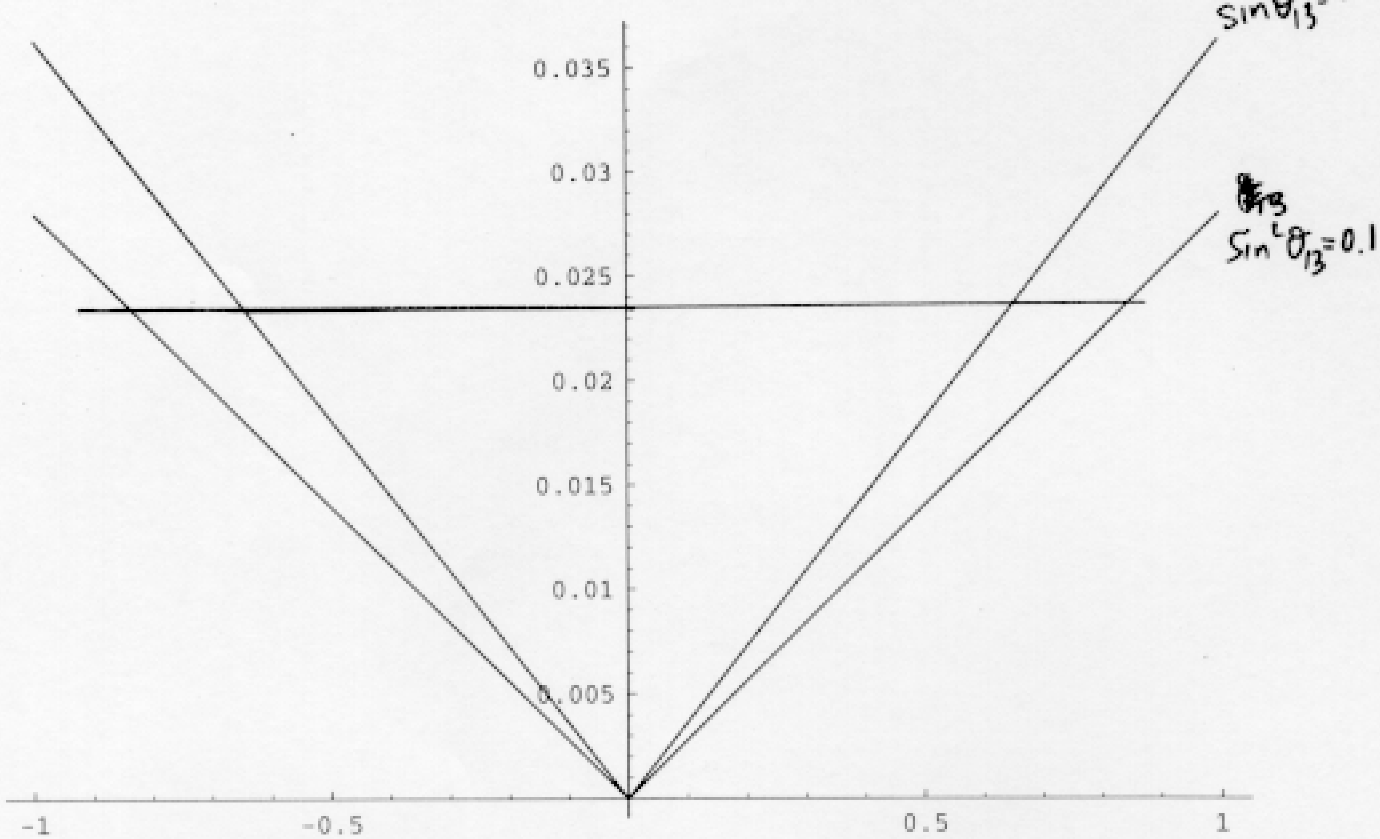
$E_\mu = 5.5$ GeV

$N_{\nu_\mu cc} = 6000$ (from Bueno et al.)

Only statistical errors

Sensitivity to θ_{13}

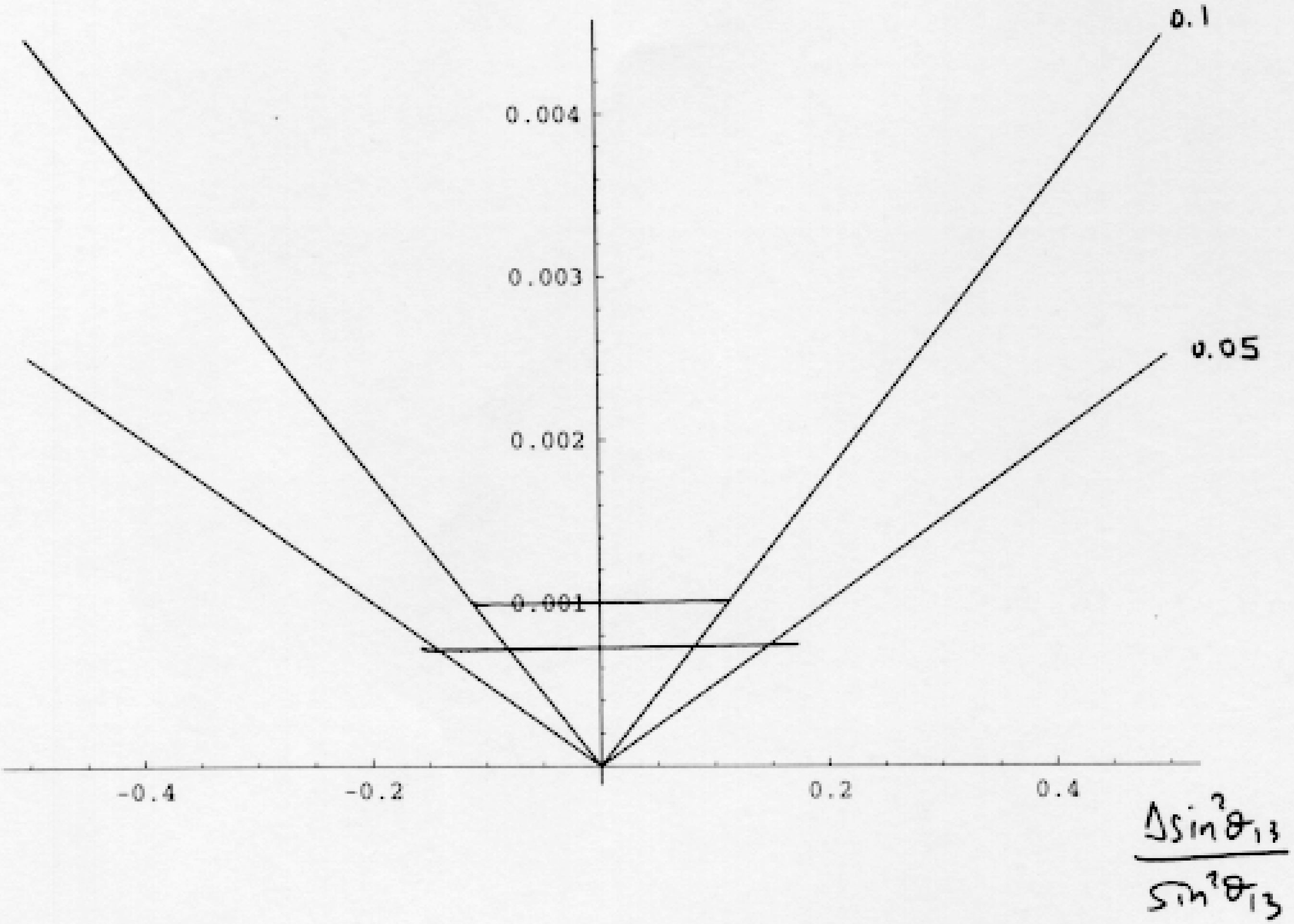
N_e/N_μ no charge id.



$\theta_{23} = 40°$

$\Delta m^2_{23} = 10^{-3}$

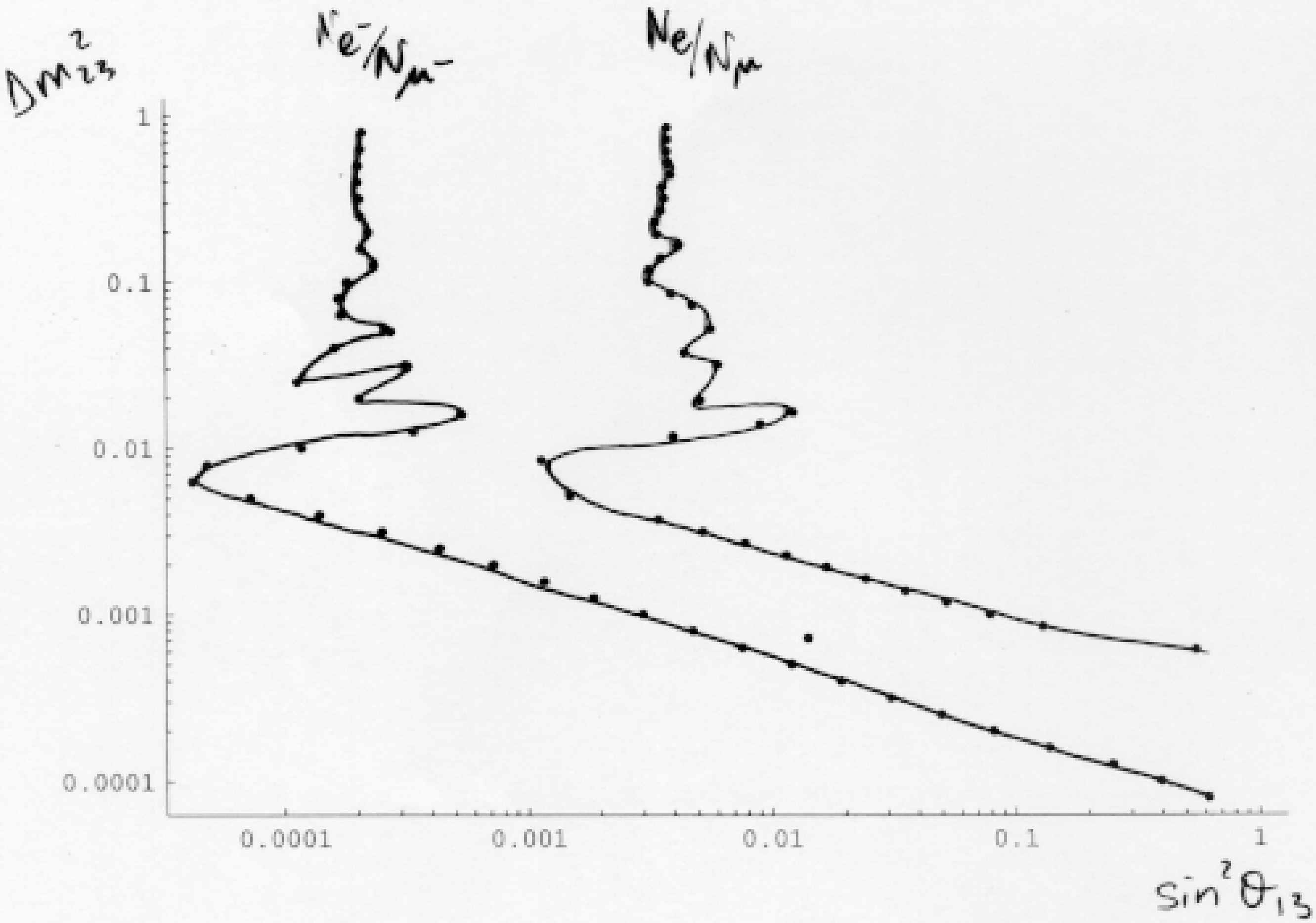
Sensitivity to ϑ_{13} $\quad \frac{N_{e^-}}{N_{\mu^-}}$



$\vartheta_{23} = 40^\circ$

$\Delta m^2_{23} = 10^{-3}$

$\vartheta_{23} = 40°$



Only statistical errors

4.8 kTon (from A. Bueno et al.)

$10^{21}\ \mu$

CP VIOLATION

How large?

Where to look?

Majorana phases unobservable $\propto \frac{M_{\nu_i}}{E}$

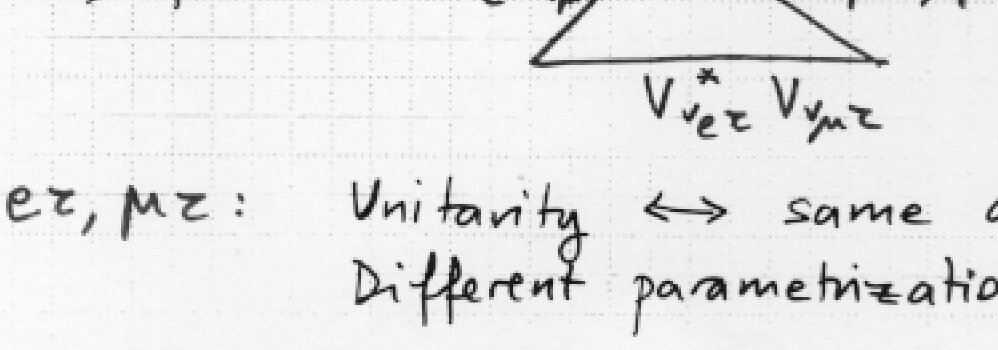
Dirac phase "δ"

$$\left.\begin{aligned} A_{\cancel{CP}} &= \frac{P(\nu_i \to \nu_j) - P(\bar{\nu}_i \to \bar{\nu}_j)}{+} \\ A_T &= \frac{P(\nu_i \to \nu_j) - P(\nu_j \to \nu_i)}{+} \end{aligned}\right\}$$ Equivalent in vacuum assuming CPT

Angle Dependence $\Leftrightarrow$ unitarity triangles

$$V = \begin{pmatrix} V_{\nu_e e} & V_{\nu_e \mu} & V_{\nu_e \tau} \\ V_{\nu_\mu e} & V_{\nu_\mu \mu} & V_{\nu_\mu \tau} \\ V_{\nu_\tau e} & V_{\nu_\tau \mu} & V_{\nu_\tau \tau} \end{pmatrix} \qquad V^+ V = \underline{1}$$

$e\mu$: $\quad V_{\nu_e e}^* V_{\nu_\mu e} + V_{\nu_e \mu}^* V_{\nu_\mu \mu} + V_{\nu_e \tau}^* V_{\nu_\mu \tau} = 0$



$e\tau, \mu\tau$: Unitarity $\leftrightarrow$ same area
Different parametrizations $\to$ rotated triangles

Angle Dependence

CP odd amplitudes $\propto$ H = area of triangles

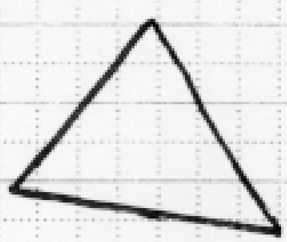
$$= -\frac{1}{2}\,\mathrm{Im}\left\{ V^*_{\nu_e e}\, V_{\nu_\mu e} \left(V^*_{\nu_e \mu}\, V_{\nu_\mu \mu} \right)^* \right\}$$

$$= \frac{1}{8}\, c_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \sin\delta$$

CP even amplitudes $\propto$ sides of triangles

$$\left(|V^*_{\nu_e e}\, V_{\nu_\mu e}|,\ |V^*_{\nu_e \mu}\, V_{\nu_e \mu}|, \text{etc} \right)$$

$$A_{\not{CP}} = \frac{\text{CP odd}}{\text{CP even}}$$



$A_{\not{CP}} = O(1)$

$A_{\not{CP}} \ll 1.$

Mass dependence

$$A_{\not{CP}} = 0 \quad \text{if} \quad \Delta m_{ij}^2 = 0 \quad \text{any } i \neq j$$

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= -4 \sum_{k>j} \mathrm{Re}\left[V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k} \right] \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E} \right) \\ \bar{\nu}_\alpha \to \bar{\nu}_\beta) & \\ & \underbrace{\pm \frac{1}{2} \sum_{k>j} \mathrm{Im}\left[V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k} \right] \sin\left(\frac{\Delta m_{kj}^2 L}{2E} \right)}_{\propto} \end{aligned}$$

$$H \cdot \Delta m_{12}^2 \, \Delta m_{13}^2 \, \Delta m_{23}^2 \left(\frac{L}{2E} \right)^3$$

$\not{CP}$ will be larger at

$$E/L \sim \mathrm{Min}\left(\Delta m_{ij}^2 \right)$$

At atmospheric distances

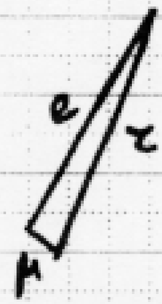
$$A_{\not{CP}} \sim \Delta m_{12}^2 \frac{L}{2E} \quad \text{at least!}$$

eμ $\qquad V^*_{\nu_e e} V_{\nu_\mu e}$, $V^*_{\nu_e \mu} V_{\nu_\mu \mu}$, $V^*_{\nu_e \tau} V_{\nu_\mu \tau}$ $\qquad A$

$\sin^2\theta_{12} = 5 \cdot 10^{-3}$

$\theta_{13} = 13^\circ$

$\Delta m^2_{12} = 10^{-4} - 10^{-5}$



$< 8\, 10^{-3}$

$\sin^2\theta_{12} = 0.5$

$\theta_{13} = 13^\circ$

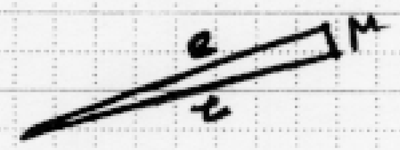
$\Delta m^2_{12} = 10^{-4} - 10^{-5}$

$< 8 \cdot 10^{-2}$

$\sin^2\theta_{12} = 10^{-2}$

$\theta_{13} = 45^\circ$

$\Delta m^2_{12} = 10^{-5} - 10^{-6}$

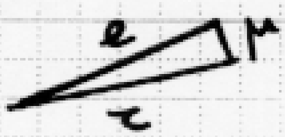


$< 3.6 \cdot 10^{-4}$

$\sin^2\theta_{12} = 10^{-4}$

$\theta_{13} = 45^\circ$

$\Delta m^2_{12} = 10^{-5} - 10^{-6}$



$< 1.2 \cdot 10^{-3}$

$\Delta m^2_{23} = 10^{-3} \qquad \theta_{23} = 45^\circ$