A theoretical study of the parameter degeneracy at future neutrino experiments

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from hep-ph/0312072

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· Cern, 18th February 2004 ·

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1 Introduction

Experimental situation

- solar ν's + CHOOZ + KamLAND $\quad \left\{ \begin{array}{c} \Delta m^2_{12} \sim 7 \cdot 10^{-5} \text{ eV}^2 \\ \sin^2 2\theta_{12} \sim 0.82 \end{array} \right.$

- CHOOZ + SuperK $\quad \left\{ \begin{array}{c} |\Delta m^2_{23}| \sim 3 \cdot 10^{-3} \text{ eV}^2 \\ \sin^2 2\theta_{23} > 0.9 \end{array} \right.$

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BUT

$\sin^2\theta_{13} < 0.03 \longrightarrow \theta_{13}$ **poorly known**

δ **completely unknown** $\longrightarrow$ **leptonic CP-violation ?**

can we measure them?

no because of DEGENERACIES !!!

- strong correlation between θ_{13} and δ in the transition probabilities: two different couples of (θ_{13},δ) give the same probabilities for ν and $\bar{\nu}$

$$\left.\begin{array}{l} \textbf{"true" value : chosen by Nature} \\ \textbf{"false" value : the clone} \end{array}\right\} \rightarrow$$ *intrinsic* ambiguity

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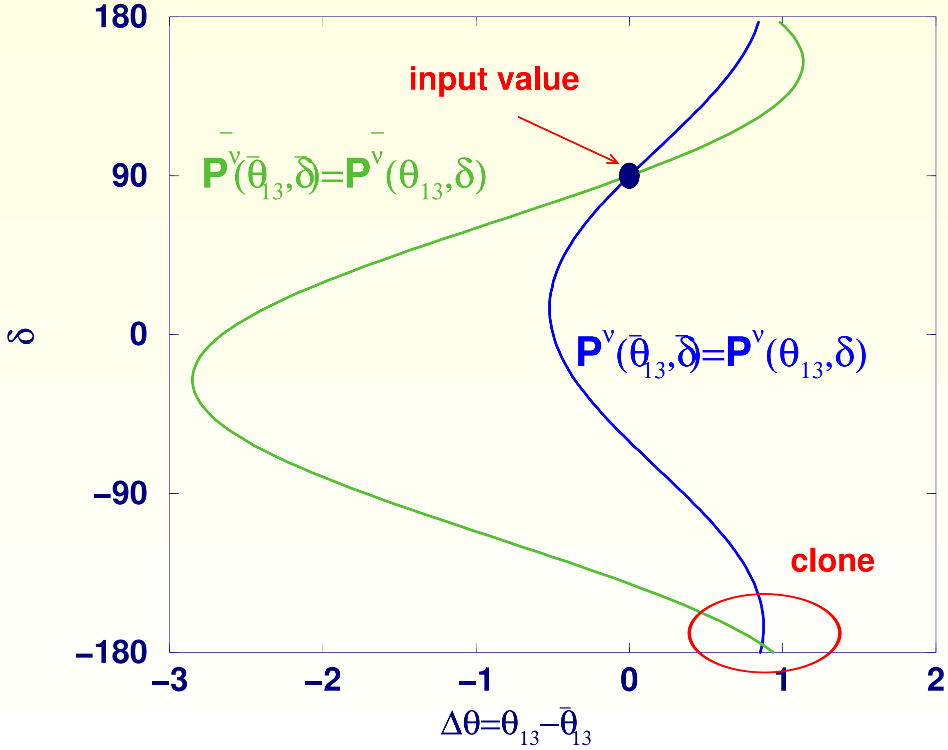

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- mirrors from $s_{atm} = \pm 1, s_{oct} = \pm 1$ (mixed ambiguity)

eightfold degeneracy

How to calculate the location of the clones?

equiprobability curves

$$\underbrace{P^{\pm}_{\alpha\beta}(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct})}_{\text{"}true\ probability\text{"}} = \underbrace{P^{\pm}_{\alpha\beta}(\theta_{13}, \delta; s_{atm}, s_{oct})}_{\text{"}theoretical\ prediction\text{"}}$$

How to calculate the location of the clones?

equal number of events equations (ENE)

$$\underbrace{N^{i}_{l\pm}(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct})}_{\text{"experimental result"}} = \underbrace{N^{i}_{l\pm}(\theta_{13}, \delta; s_{atm}, s_{oct})}_{\text{"theoretical prediction"}}$$

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depending of the ambiguity considered:

intrinsic ambiguity

$$N(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N(\theta_{13},\delta;s_{atm}=\bar{s}_{atm},s_{oct}=\bar{s}_{oct})$$

sign ambiguity

$$N(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N(\theta_{13},\delta;s_{atm}=-\bar{s}_{atm},s_{oct}=\bar{s}_{oct})$$

octant ambiguity

$$N(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N(\theta_{13},\delta;s_{atm}=\bar{s}_{atm},s_{oct}=-\bar{s}_{oct})$$

mixed ambiguity

$$N(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N(\theta_{13},\delta;s_{atm}=-\bar{s}_{atm},s_{oct}=-\bar{s}_{oct})$$

$$\downarrow$$

implicit equation in δ:

$$F^{\pm}(\delta) = G^{\pm}(\theta_{13}, \bar{\theta}_{13}, \bar{\delta})$$

from which we can numerically extract:

$$\sin^2 2\theta_{13} \qquad \text{and} \qquad \delta$$

$$\downarrow$$

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Results presented in the plane

$$\Delta\theta_{13} = \underbrace{\theta_{13}}_{\textit{clone solution}} - \underbrace{\bar{\theta}_{13}}_{\textit{true solution}}$$

$$\delta = F^{-1}[G^{\pm}(\theta_{13}, \bar{\theta}_{13}, \bar{\delta})]$$

FLOW OF DEGENERACIES

2 Theoretical flow of degeneracies

we consider different experimental set-ups:

Neutrino Factory looking at the $\nu_e \rightarrow \nu_\mu$ (NFG-L=2810 Km)

Neutrino Factory looking at the $\nu_e \rightarrow \nu_\tau$ (NFS-L=732 Km)

SuperBeam facilities looking at the $\nu_\mu \rightarrow \nu_e$ (SB-L=130 Km)

β beam facilities looking at the $\nu_e \rightarrow \nu_\mu$ (BB-L=130 Km, $\gamma_{6_{He}} = 60$, $\gamma_{18_{Ne}} = 100$)

3 Analitical treatment of the ambiguities

to start speaking... $\nu_e \rightarrow \nu_\mu$

$$P^{\pm}_{e\mu}(\theta_{13},\delta) = X_{\pm}\sin^2(2\,\theta_{13}) +$$

$$\left[Y_{\pm}\,\cos\left(\frac{\Delta_{atm}L}{2}\right)\,\cos\delta \pm Y_{\pm}\,\sin\left(\frac{\Delta_{atm}L}{2}\right)\,\sin\delta\right]\cos(\theta_{13})\sin(2\,\theta_{13}) + Z$$

but in a given detector: $\left\{\begin{array}{l} \nu\,\mathbf{N}\rightarrow \mathbf{l}^{-}\,\mathbf{N}^{`} \\ \bar{\nu}\,\mathbf{N}\rightarrow \mathbf{l}^{+}\,\mathbf{N}^{`} \end{array}\right.$ $\rightarrow$ number of events

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$$N^i_{l\pm}(\theta_{13},\delta) \quad = \quad \left\{\frac{d\sigma_{\nu_\mu(\bar\nu_\mu)}}{dE_\mu}\,\otimes\,P^{\pm}_{e\mu}\,\otimes\,\frac{d\Phi_{\nu_e(\bar\nu_e)}}{dE_\nu}\right\}_{E_i}^{E_i+\Delta E_\mu}$$

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to start speaking...$\nu_e \to \nu_\mu$

$$P^{\pm}_{e\mu}(\theta_{13},\delta) = X_\pm \sin^2(2\,\theta_{13}) +$$

$$\left[Y_\pm \cos\left(\frac{\Delta_{atm} L}{2}\right) \cos\delta \pm Y_\pm \sin\left(\frac{\Delta_{atm} L}{2}\right) \sin\delta \right] \cos(\theta_{13}) \sin(2\,\theta_{13}) + Z$$

$$N^i_{l\pm}(\theta_{13},\delta) \quad = \quad \left\{ \frac{d\sigma_{\nu_\mu(\bar\nu_\mu)}}{dE_\mu} \otimes P^{\pm}_{e\mu} \otimes \frac{d\Phi_{\nu_e(\bar\nu_e)}}{dE_\nu} \right\}_{E_i}^{E_i+\Delta E_\mu}$$

$$\left\{ \begin{array}{lll} N^i_{l-} & = & \left\{ I^+_1 \sin^2(2\theta_{13}) + \left[I^+_2 \cos\delta + I^+_3 \sin\delta \right] \cos\theta_{13} \sin(2\theta_{13}) + I_4 \right\}^i , \\ N^i_{l+} & = & \left\{ I^-_1 \sin^2(2\theta_{13}) + \left[I^-_2 \cos\delta - I^-_3 \sin\delta \right] \cos\theta_{13} \sin(2\theta_{13}) + I_4 \right\}^i . \end{array} \right.$$

theoretical flow: the intrinsic ambiguity

$$N(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N(\theta_{13}, \delta; s_{atm} = \bar{s}_{atm}, s_{oct} = \bar{s}_{oct})$$

theoretical flow: the intrinsic ambiguity

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in the vacuum approx

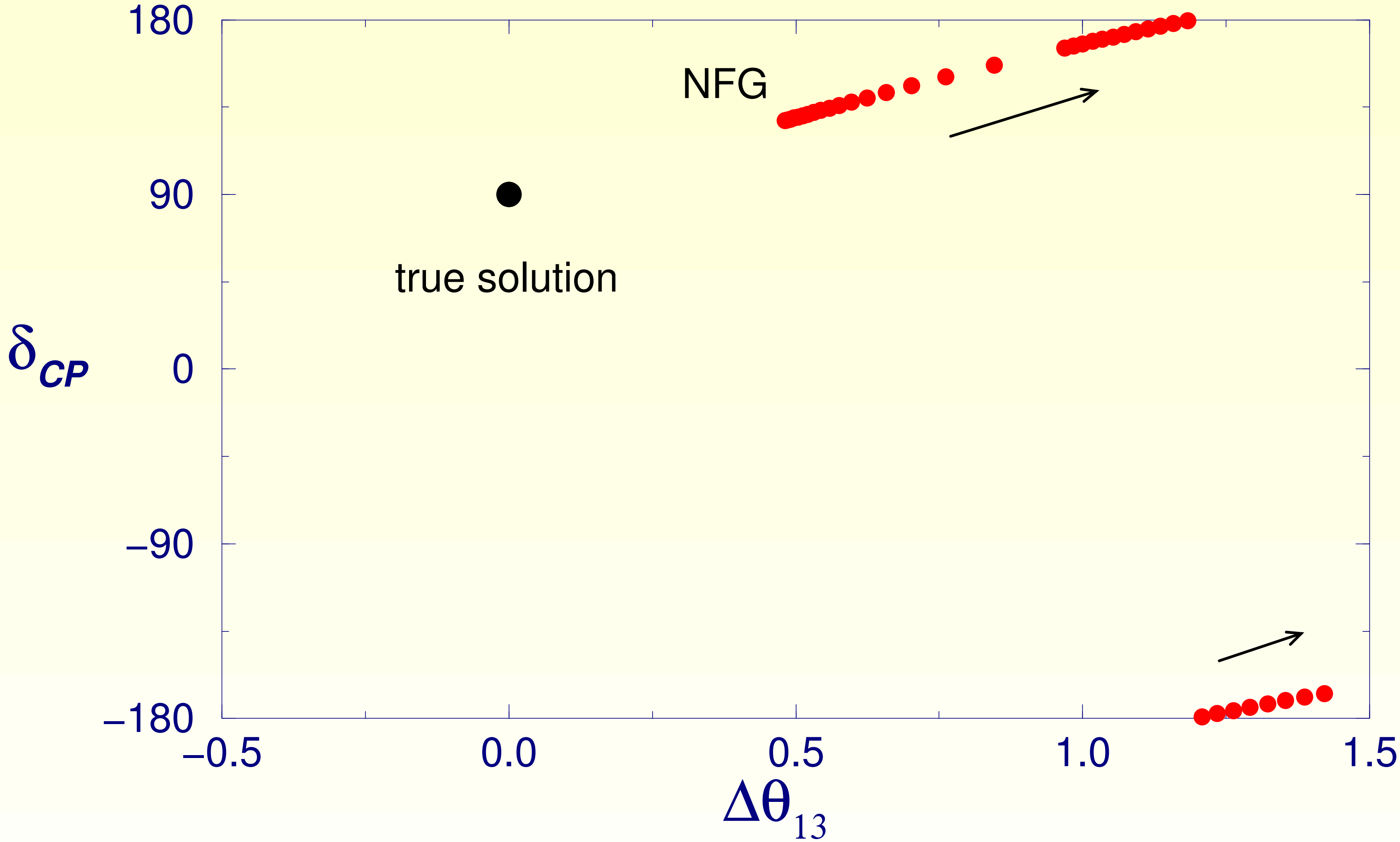
$$\sin^2 2\bar{\theta}_{13} \qquad \text{(true solution)}$$

$$\sin^2 2\theta_{13} = \underbrace{\sin^2 2\bar{\theta}_{13} + \left(\frac{\mathrm{I}_2}{\mathrm{I}_1}\right)^2 \left[1 + 2\left(\frac{\mathrm{I}_1}{\mathrm{I}_2}\right)\cos\bar{\delta}\,\sin 2\bar{\theta}_{13}\right]}_{\textbf{clone solution!}}$$

Note:

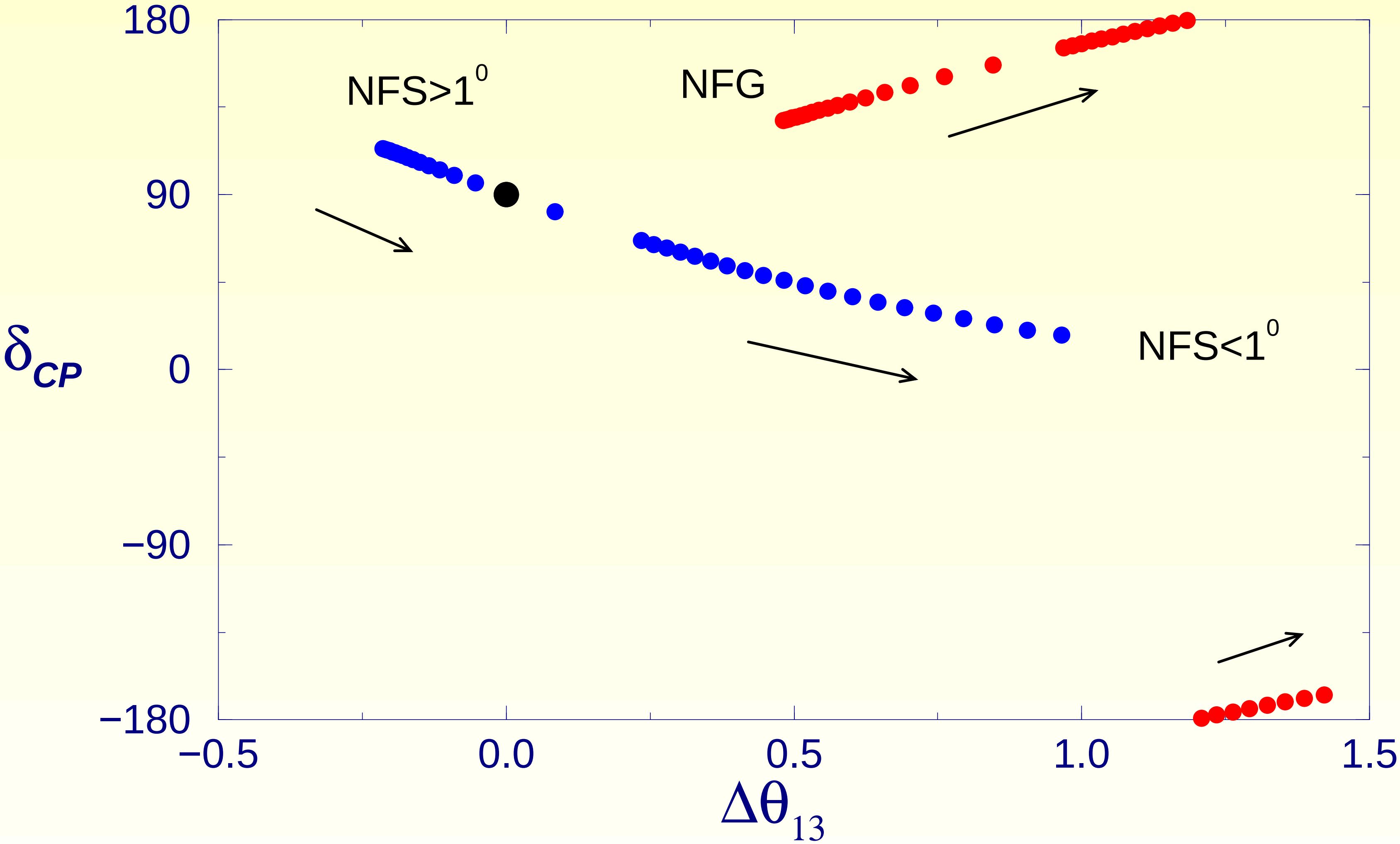
- $\theta_{13} = \bar{\theta}_{13}$ even in the full computation
- $\sin^2 2\theta_{13} \geq \sin^2 2\bar{\theta}_{13}$ for any value of θ_{13} (for $\nu_e \to \nu_\mu$)
- $I_2^\tau = -I_2^\mu \to \theta_{13}-$shift can be either positive or negative

theoretical flow: the intrinsic ambiguity



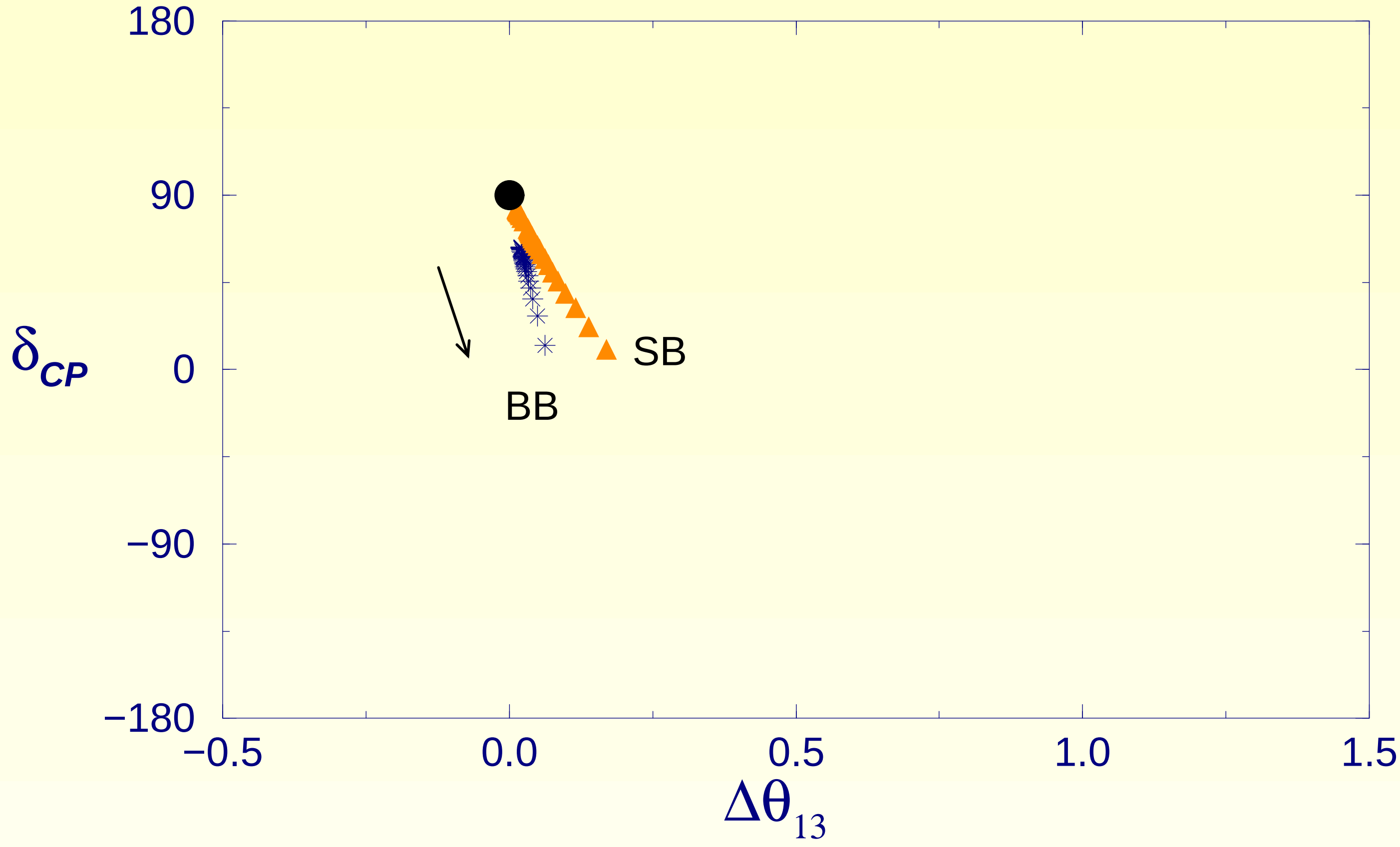
decreasing θ_{13} from 10^o to 0.1^o

theoretical flow: the intrinsic ambiguity



VERY WELL SEPARATED FLOWS!

theoretical flow: the intrinsic ambiguity



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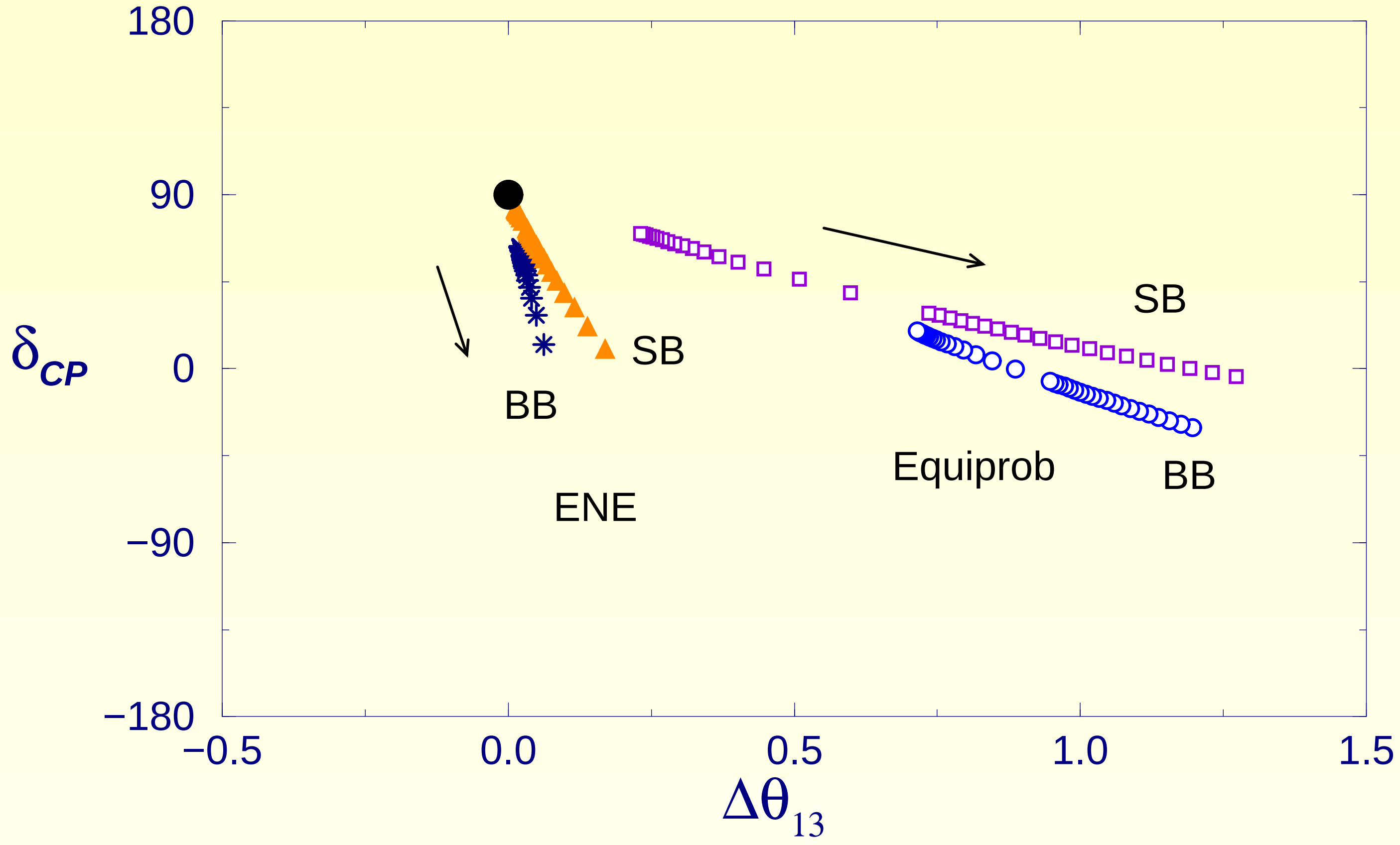

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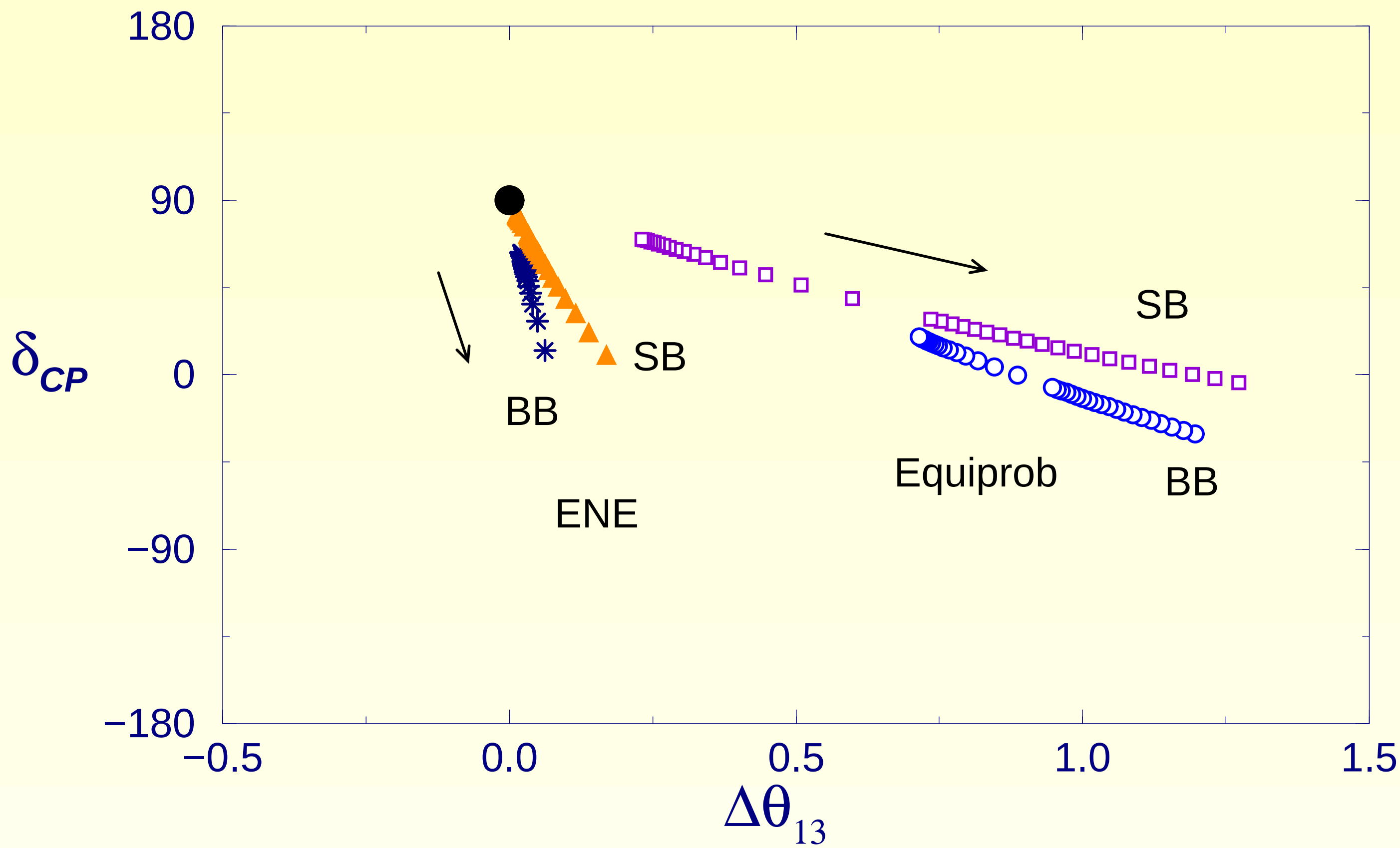
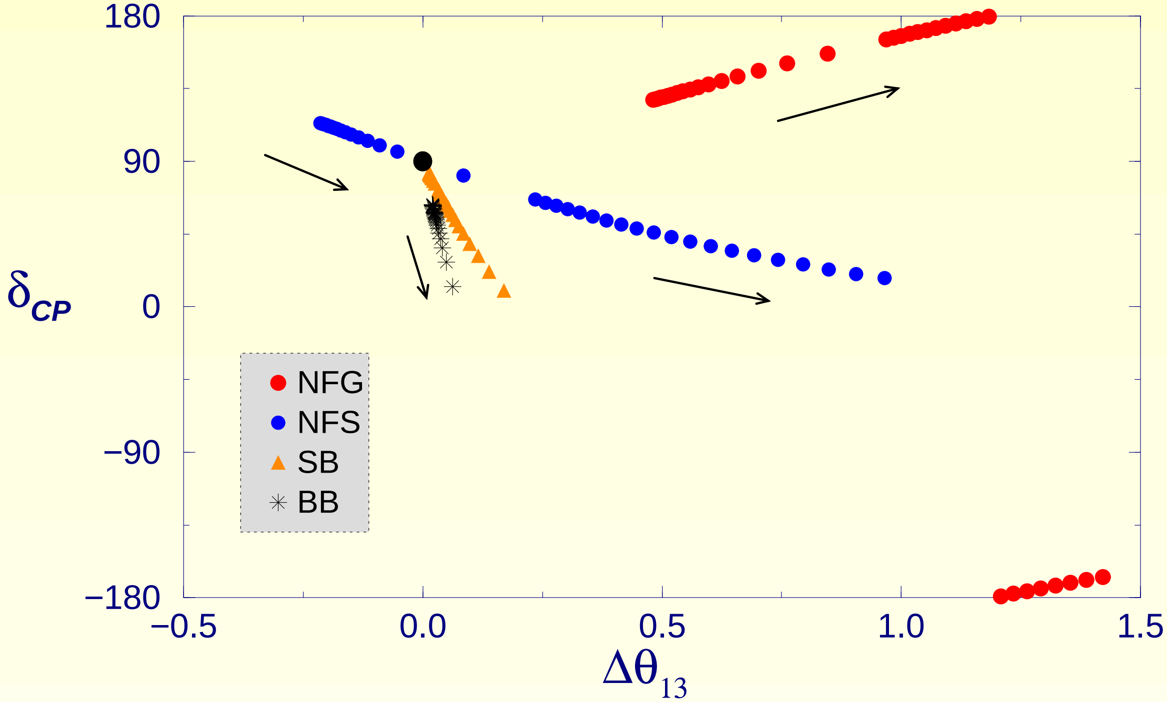


- very different ENE and Equiprob patterns $\longrightarrow$ wrong conjectures!
- practically identical ENE flows
- very small $\Delta\theta_{13} \longrightarrow$ $\begin{cases} \textbf{clones do not interfere with the measure of } \theta \\ \textbf{reduced ability of measuring the CP phase} \end{cases}$

Putting all together



decide the most

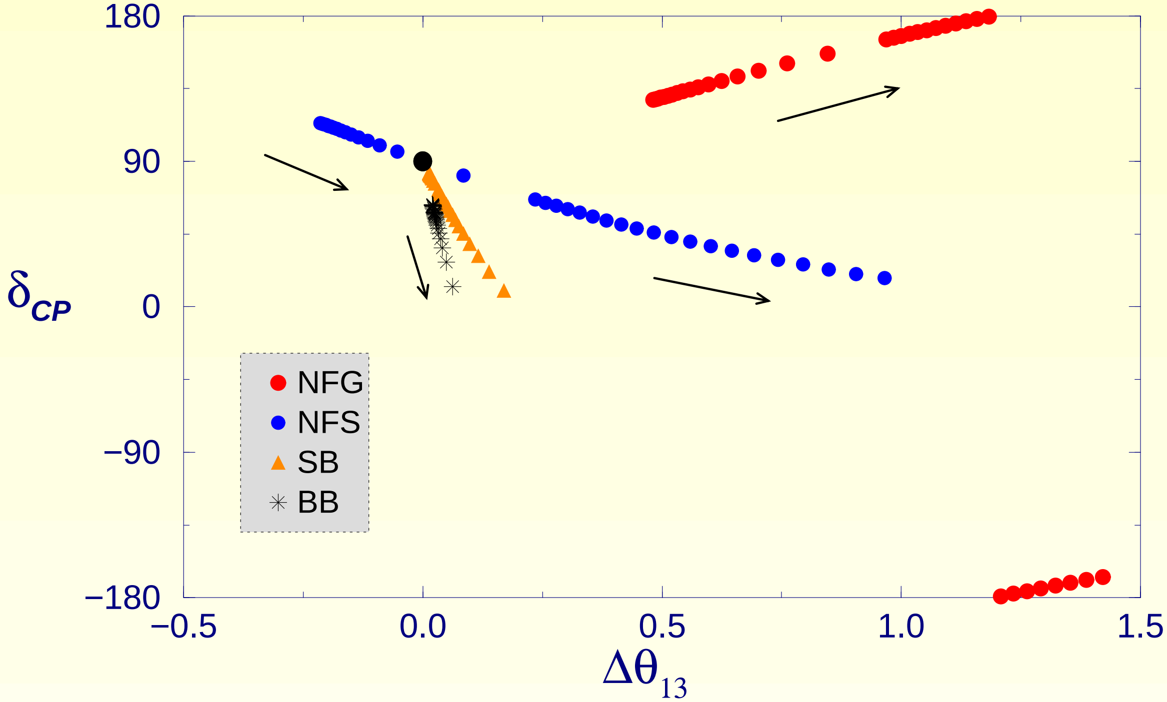
promising

combination !!!

↓

discarding set-ups adding coherently

Putting all together



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discarding set-ups adding coherently

NFG + NFS and/or SB/BB

could solve the intrinsic ambiguity

theoretical flow: the sign ambiguity

$$N(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N(\theta_{13}, \delta; s_{atm} = -\bar{s}_{atm}, s_{oct} = \bar{s}_{oct})$$

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- two branches of solutions $\rightarrow \sin^2 2\theta_{13} \neq \sin^2 2\bar{\theta}_{13}$

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- two branches of solutions $\rightarrow \sin^2 2\theta_{13} \neq \sin^2 2\bar{\theta}_{13}$
- vacuum limit:

$$(\sin^2 2\theta_{13})_{sign} = (\sin^2 2\theta_{13})_{int}$$

$$\delta_{\rm sign} = \pi - \delta_{\rm int}$$

theoretical flow: the sign ambiguity

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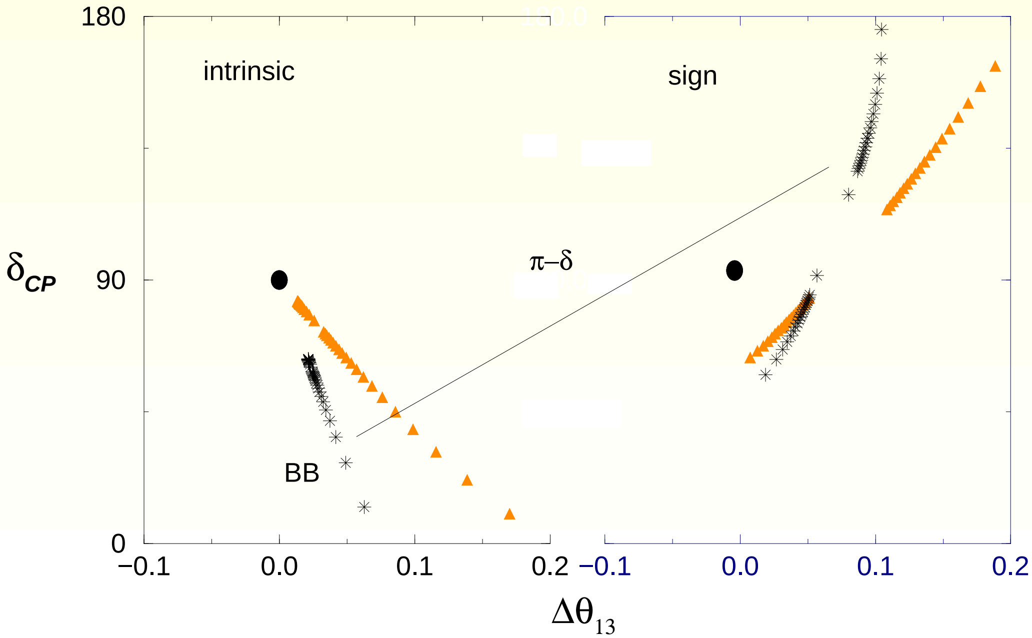
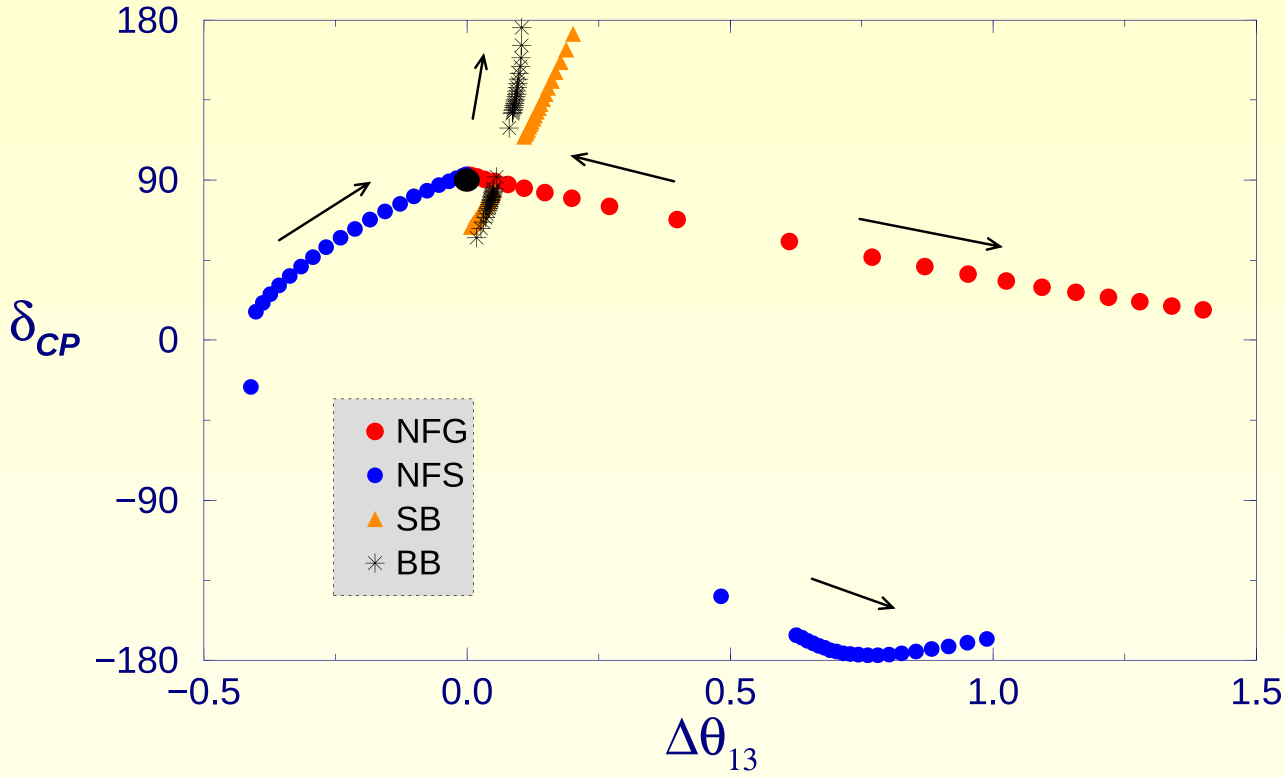
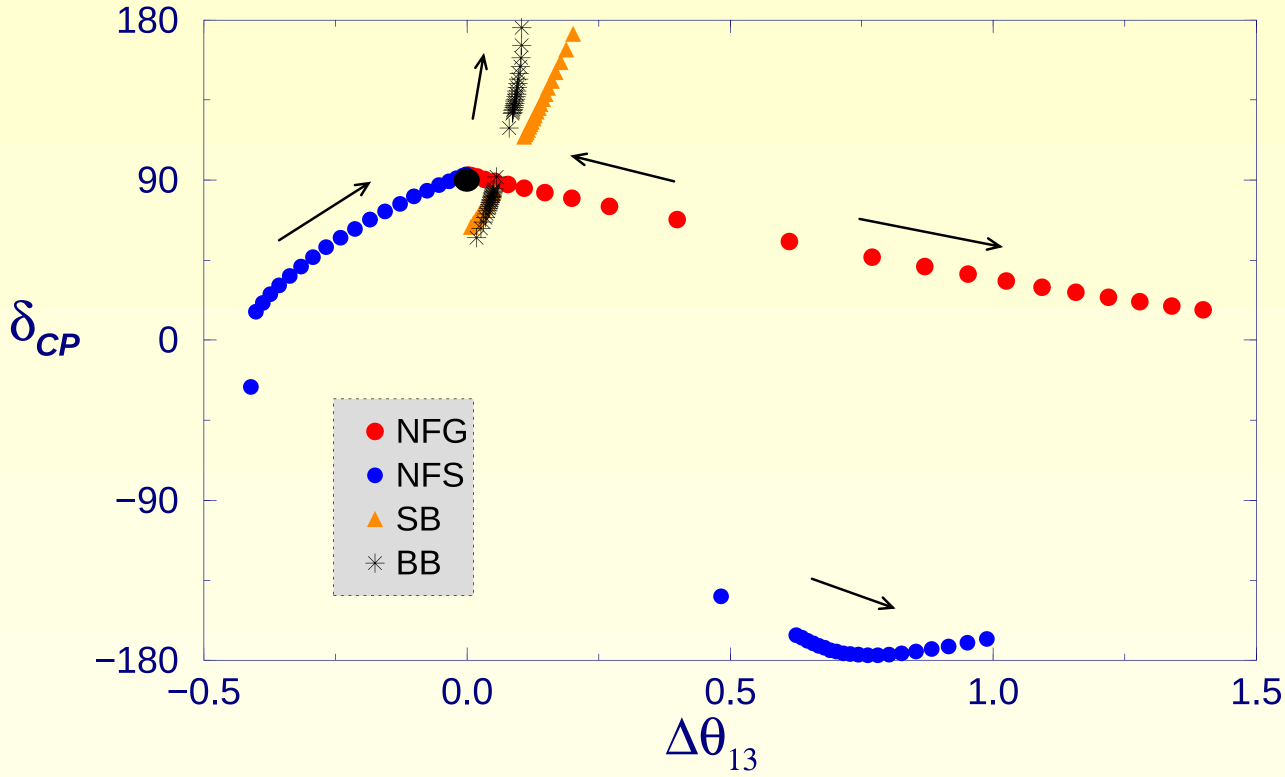


theoretical flow: the sign ambiguity



- SB and BB flows very near $\longrightarrow$ not suited to solve the sign degeneracy
- NFG and NFS flows are quite separated
- superposition between the four flows but close to the "true" point

theoretical flow: the sign ambiguity



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could solve the sign ambiguity

theoretical flow: the octant-mixed ambiguities

octant

$$N(\bar{\theta}_{13}, \bar{\delta}; \bar{s}_{atm}, \bar{s}_{oct}) = N(\theta_{13}, \delta; s_{atm} = \bar{s}_{atm}, s_{oct} = -\bar{s}_{oct})$$

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important features:

- two different branches of solutions in both cases
- in vacuum:

  $\delta_{\rm mixed} = \pi - \delta_{\rm oct}$

  $(\sin^2 2\theta_{13})_{mixed} = (\sin^2 2\theta_{13})_{oct}$

theoretical flow: the octant ambiguity

a very complicated pattern of the solutions appears

$\downarrow$

example for $\bar{\theta}_{13} = 1^o$, $\bar{\delta} = 90^o$ and $\theta_{23} = 40^o$

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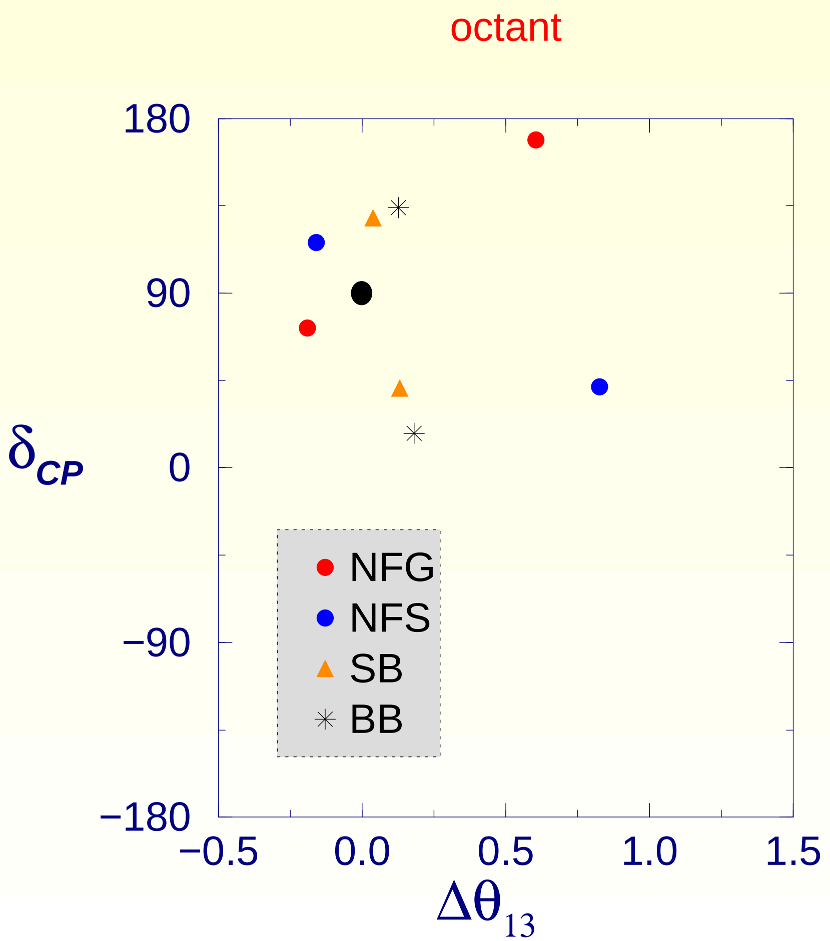

two solutions for each facility

SB and BB flows very close

substantial overlap of clones

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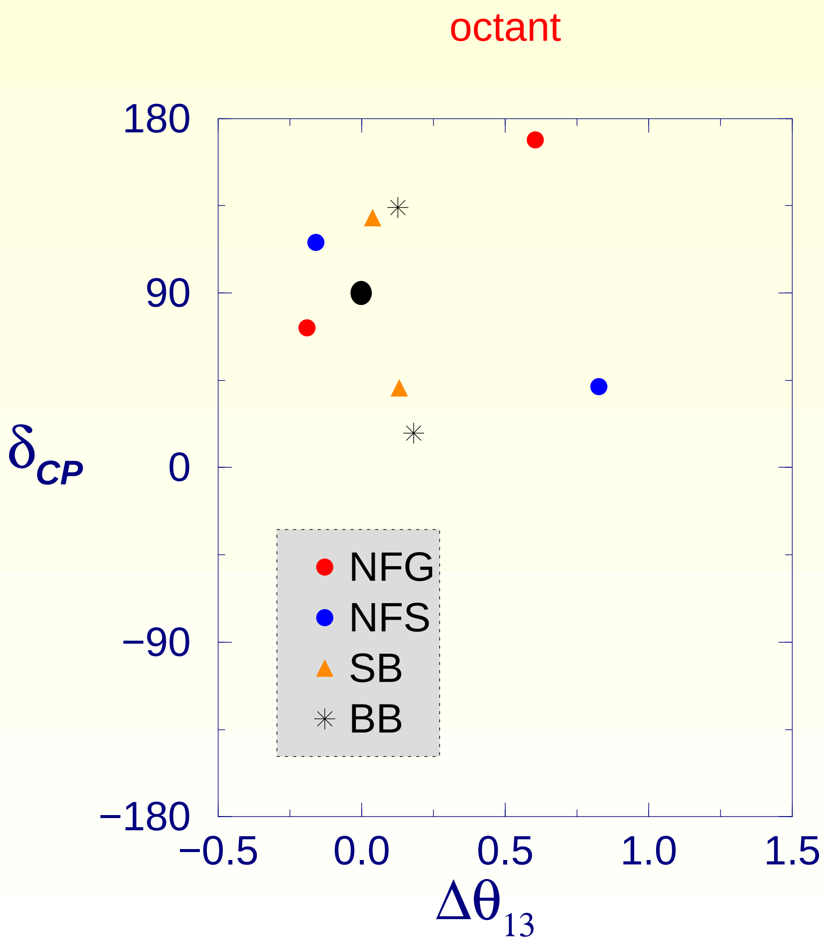

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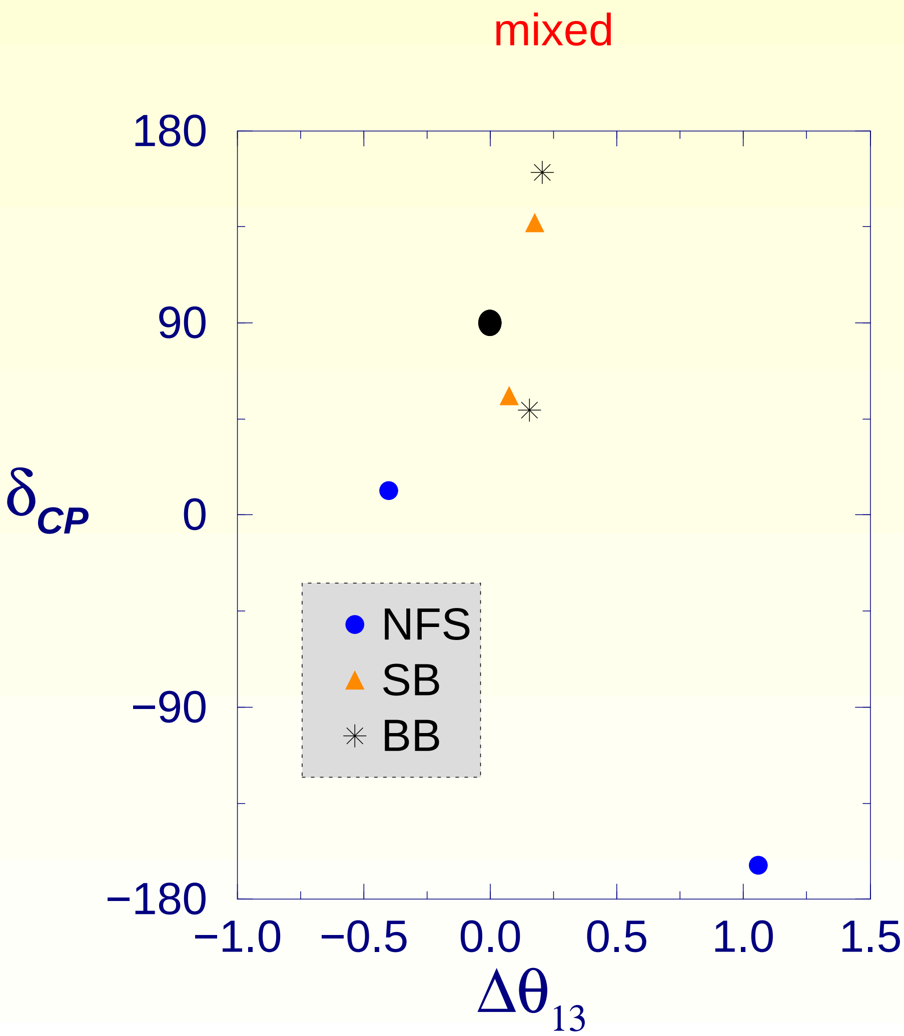

no NFG clones for these parameters

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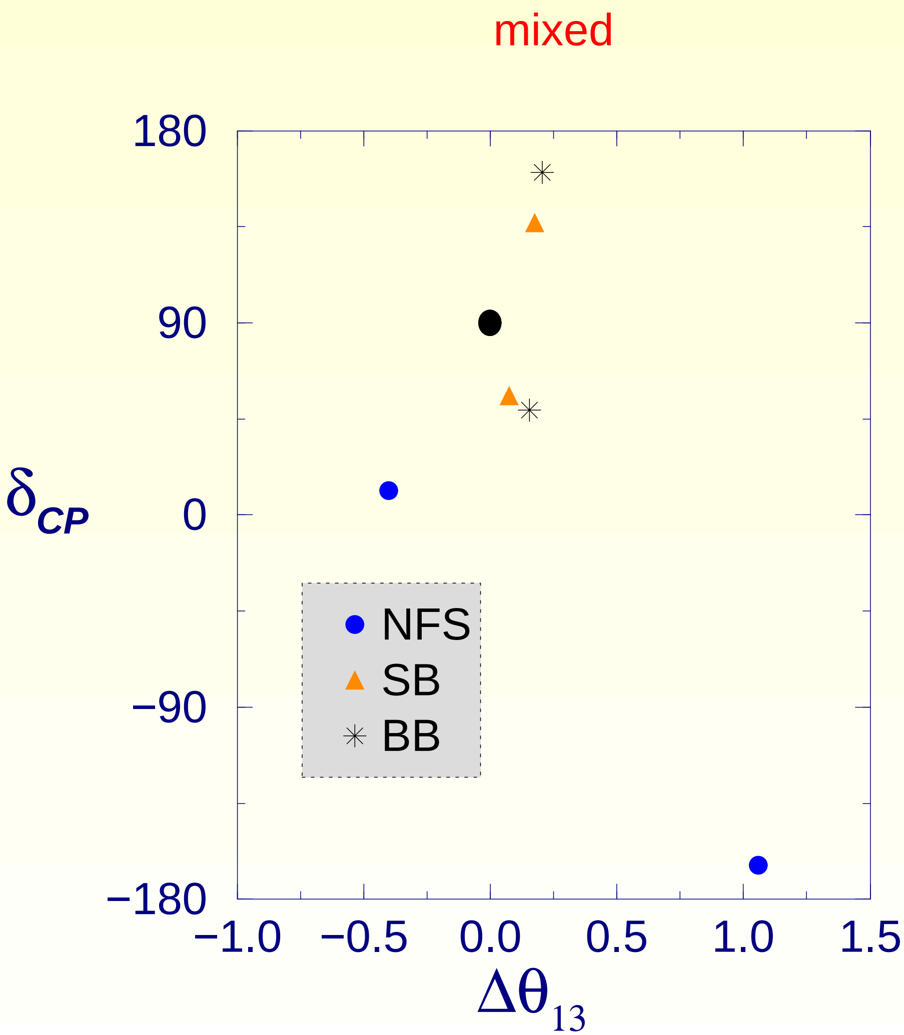

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NFG and/or NFS + SB/BB

could solve the mixed ambiguity

Conclusions

1. we presented the general technique to calculate the clone location
   - computed from ENE or equiprobability equations
   - we adopt the first strategy to avoid wrong conjectures
2. we analized the behaviour of clones for different beams
   - SuperBeams and BetaBeams do not help in solving ANY of the degeneracies
   - The NFG and Silver clones lie well apart for $\theta_{13} > 1^o$
3. numerical studies indicate the combination to eliminate clones
   - NFG + NFS and/or BB/SB could in principle solve the intrinsic, sign and mixed ambiguities
   - NFG + NFS + BB/SB could in principle solve the octant ambiguity
4. warning...
   - results must be confirmed by taking into account statistics and systematics of a given experimental combination

an example: the intrinsic ambiguity

How to calculate the location of the clones?

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from $N(\bar{\theta}_{13}, \bar{\delta}) = N(\theta_{13}, \delta) \rightarrow$ implicit equation in δ:

$$F^{\pm}(\delta) = G^{\pm}(\theta_{13}, \bar{\theta}_{13}, \bar{\delta})$$

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$$F^{\pm}(\delta) = \cos\delta \pm \left(\frac{I_3^{\pm}}{I_2^{\pm}}\right) \sin\delta$$

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$$F^{\pm}(\delta) = \cos\delta \pm \left(\frac{I_3^{\pm}}{I_2^{\pm}}\right)\sin\delta$$

$$G^{\pm}(\theta_{13}, \bar{\theta}_{13}, \bar{\delta}) = \left(\frac{I_1^{\pm}}{I_2^{\pm}}\right) f(\theta_{13}, \bar{\theta}_{13}) + F^{\pm}(\bar{\delta})\, g(\theta_{13}, \bar{\theta}_{13})$$

an example: the intrinsic ambiguity

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$$F^{\pm}(\delta) = \cos\delta \pm \left(\frac{I_3^{\pm}}{I_2^{\pm}}\right)\sin\delta$$

$$G^{\pm}(\theta_{13},\bar{\theta}_{13},\bar{\delta}) = \left(\frac{I_1^{\pm}}{I_2^{\pm}}\right) f(\theta_{13},\bar{\theta}_{13}) + F^{\pm}(\bar{\delta})\, g(\theta_{13},\bar{\theta}_{13})$$

$$\begin{cases} f(\theta_{13},\bar{\theta}_{13}) & = & \dfrac{\sin^2(2\bar{\theta}_{13}) - \sin^2(2\theta_{13})}{\cos\theta_{13}\sin(2\theta_{13})}\,, \\ g(\theta_{13},\bar{\theta}_{13}) & = & \dfrac{\cos\bar{\theta}_{13}\sin(2\bar{\theta}_{13})}{\cos\theta_{13}\sin(2\theta_{13})} \end{cases}$$

theoretical flow: the intrinsic ambiguity

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$$\sin\delta = C(L)\, f(\theta_{13},\bar{\theta}_{13}) + g(\theta_{13},\bar{\theta}_{13})\,\sin\bar{\delta}$$

$$\cos\delta = D(L)\, f(\theta_{13},\bar{\theta}_{13}) + g(\theta_{13},\bar{\theta}_{13})\,\cos\bar{\delta}$$

$$C = \left[\frac{I_1^+ I_2^- - I_1^- I_2^+}{I_3^+ I_2^- + I_3^- I_2^+}\right] \qquad D = \left[\frac{I_1^+ I_3^- + I_1^- I_3^+}{I_3^+ I_2^- + I_3^- I_2^+}\right]$$

theoretical flow: the intrinsic ambiguity

$$\sin\delta \quad = \quad C(L)\, f(\theta_{13},\bar\theta_{13}) + g(\theta_{13},\bar\theta_{13})\, \sin\bar\delta$$

$$\cos\delta \quad = \quad D(L)\, f(\theta_{13},\bar\theta_{13}) + g(\theta_{13},\bar\theta_{13})\, \cos\bar\delta$$

$$C = \left[\frac{I_1^+ I_2^- - I_1^- I_2^+}{I_3^+ I_2^- + I_3^- I_2^+}\right] \qquad D = \left[\frac{I_1^+ I_3^- + I_1^- I_3^+}{I_3^+ I_2^- + I_3^- I_2^+}\right]$$

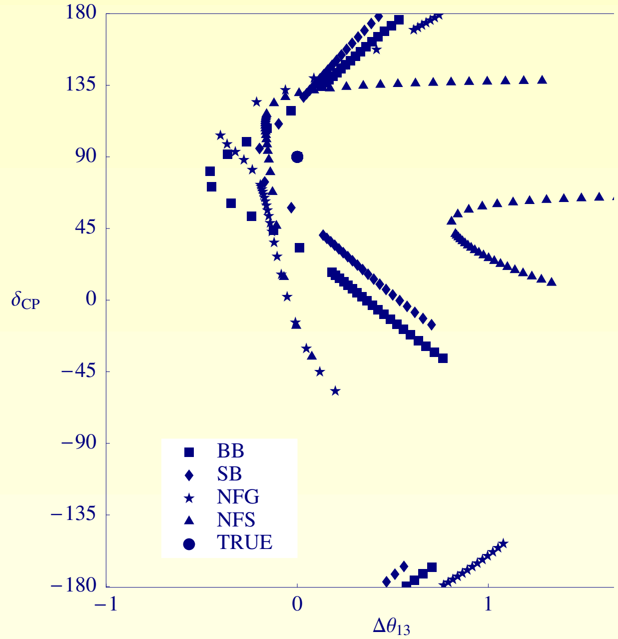
$$\sin^2 2\bar\theta_{13} \qquad \text{(true solution)}$$

$$\sin^2 2\theta_{13} = \underbrace{\sin^2 2\bar\theta_{13} + \frac{1 + 2\left[D\cos\bar\delta + C\sin\bar\delta\right]\sin 2\bar\theta_{13}}{C^2 + D^2}}_{\textbf{clone solution!}}$$

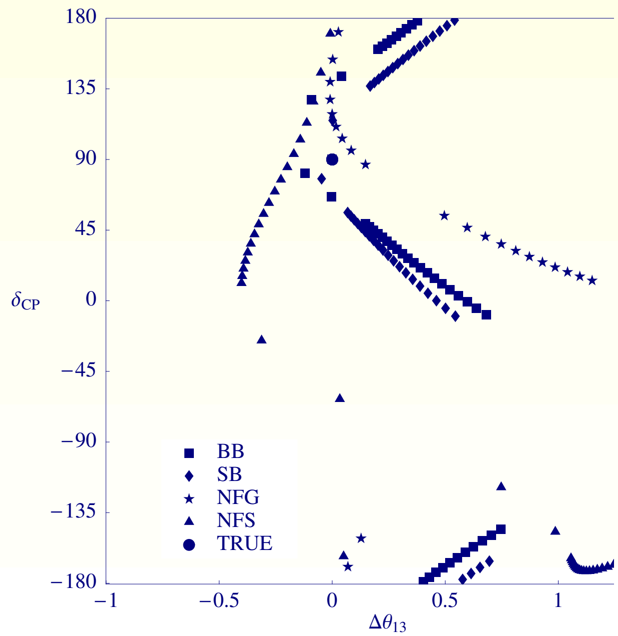
notice that for $\nu_e \to \nu_\tau$: $I_2 \to -I_2$ and $I_3 \to -I_3$

other ambiguities

octant



mixed



vacuum solutions for the mixed and octant

$$
\begin{aligned}
\sin^2 2\theta_{13} \quad = \quad & \sin^2 2\bar{\theta}_{13} + \\
& + \left\{ \tan^2\theta_{23} \, \frac{\bar{I}_2}{\bar{I}_1} \, \left[\cos\bar{\delta} \sin 2\bar{\theta}_{13} + \tan^2\theta_{23} \, \frac{\bar{I}_2}{2\bar{I}_1} \right] - \right. \\
& \left. \left(1 - \tan^2\theta_{23}\right) \left(\sin^2 2\bar{\theta}_{13} - \tan^2\theta_{23} \, \frac{\bar{I}_4}{\bar{I}_1} \right) \right\} \\
& \pm \tan^2\theta_{23} \frac{\bar{I}_2}{\bar{I}_1} \, \left\{ \left[\cos\bar{\delta} \, \sin 2\bar{\theta}_{13} + \tan^2\theta_{23} \, \frac{\bar{I}_2}{2\bar{I}_1} \right]^2 - \right. \\
& \left. \left(1 - \tan^2\theta_{23}\right) \left(\sin^2 2\bar{\theta}_{13} - \tan^2\theta_{23} \, \frac{\bar{I}_4}{\bar{I}_1} \right) \right\}^{1/2}
\end{aligned}
$$

using energy dependence of the signal

NFG and NFS