

THEORETICAL BACKGROUND TO PROTON DECAY

G. F. GIUDICE
CERN

16.01.02

- matter stability
- conservation laws
- predictions for p-decay



HENRI ANTOINE BECQUEREL



MARIE & PIERRE CURIE



ERNEST RUTHERFORD

5



HERMANN WEYL

⑥



ERNST CARL GERLACH
STÜCKELBERG



EUGENE WIGNER

QED

Local gauge invariance \Rightarrow $\left\{ \begin{array}{l} \cdot \text{massless photon} \\ \cdot \text{Coulomb's law} \end{array} \right.$

BARYON NUMBER

Global symmetry \Rightarrow p stability

B local symmetry?

- anomalous
- new force \Rightarrow difference between grav. & inertial mass

$$\alpha_B < 10^{-9} G_N m_p^2 \approx 6 \times 10^{-48}$$

Violation of B conceivable
& almost necessary!

Experiments

Early '50 $\left\{ \begin{array}{l} \tau_p > 10^{20} - 10^{22} \text{ yrs} \\ \text{"Anthropic" arguments} \end{array} \right.$

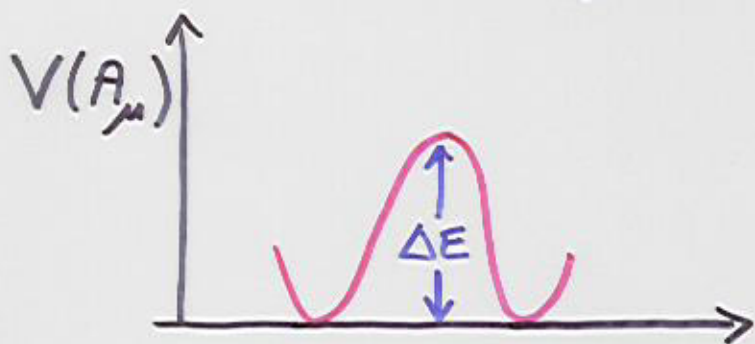
Now $\tau(p \rightarrow e^+ \pi^0) > 5 \times 10^{33} \text{ yrs}$
see GOODMAN

P-DECAY FROM ANOMALY IN SM ⁹

$$\partial_\mu J_{B+L}^\mu = 2 N_g \partial_\mu K^\mu$$

of fermion gen

Chern-Simon



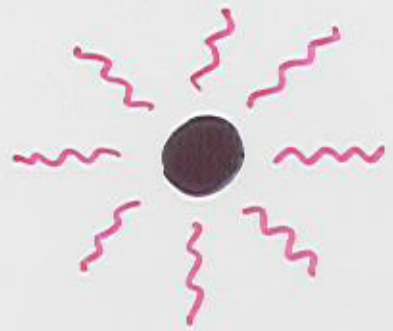
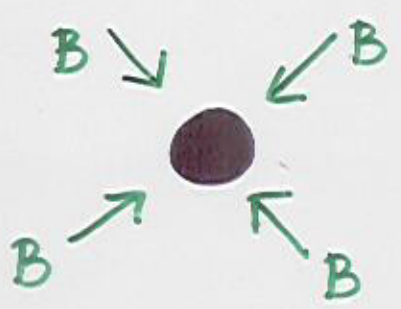
$$\text{rate} \sim \left| e^{-\frac{\Delta E \cdot \Delta t}{\hbar}} \right|^2$$

$$\frac{\Delta E \cdot \Delta t}{\hbar} \sim \frac{2\pi}{\alpha_w} \Rightarrow \text{rate} \sim 10^{-137}$$

- important conceptually
(ultimate fate of the Universe)
- early Universe
see BUCHMÜLLER

P-DECAY FROM GRAVITY

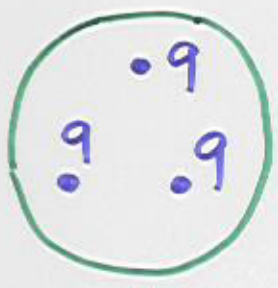
Black holes violate B



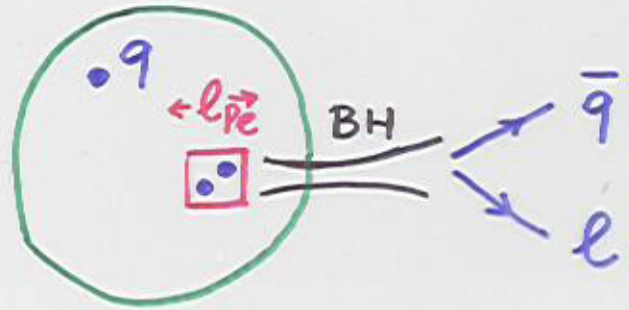
Hawking

At distances $\lesssim \frac{1}{M_{Pl}}$ \Rightarrow

space-time foam with virtual B. H.



proton



$$\tau_p \sim \frac{M_{Pl}^4}{m_p^5} \sim 10^{45} \text{ yrs}$$

Quantum-gravity effects

\Rightarrow same order of magnitude

Observable p-decay \Rightarrow
 \Rightarrow new interactions (GUTs?)

SM effective theory

$$\mathcal{L}_{\text{eff}}^{(6)} = \sum_i \frac{1}{\Lambda_i^2} \mathcal{O}_i$$

$$\mathcal{O}_1 = \bar{u}_L^c \gamma^\mu q_L \bar{d}_L^c \gamma_\mu l_L$$

$$\mathcal{O}_2 = \bar{u}_L^c \gamma^\mu q_L \bar{e}_L^c \gamma_\mu q_L$$

$$\mathcal{O}_3 = \bar{q}_{R\alpha}^c q_L^\beta \bar{q}_{R\beta}^c l_L^\alpha \quad \alpha, \beta = SU_2 \text{ indices}$$

$$\mathcal{O}_4 = \bar{d}_L^c u_R \bar{u}_L^c e_R$$

- other operators are linearly dep.
- vector particles mediate only \mathcal{O}_1 & \mathcal{O}_2

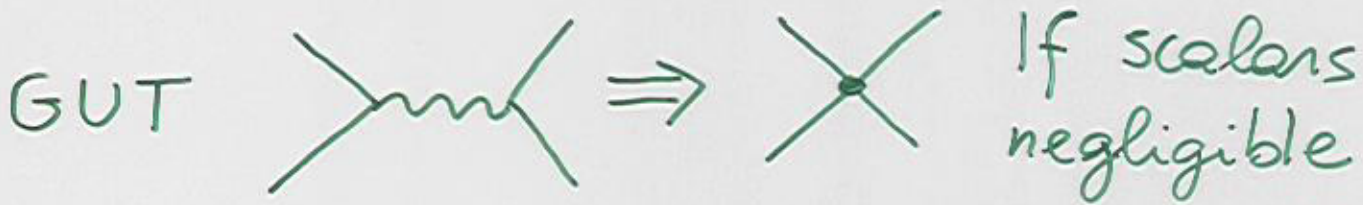


$$\Delta B = \Delta L \begin{cases} p \rightarrow e^+ X & p \rightarrow e^- X \\ p \rightarrow \bar{\nu} X & p \rightarrow \nu X \end{cases}$$

$$\frac{\Delta S}{\Delta B} = 0, -1 \begin{cases} p \rightarrow \bar{\nu} \pi^+ & p \rightarrow \bar{\nu} K^- \pi^+ \pi^+ \\ p \rightarrow \bar{\nu} K^+ & n \rightarrow e^+ K^- \end{cases}$$

$$\Delta S = 0 \Rightarrow \Delta I = \frac{1}{2} \begin{cases} \Gamma(p \rightarrow e^+ \pi^0) = \frac{1}{2} \Gamma(n \rightarrow e^+ \pi^-) \\ \Gamma(p \rightarrow \bar{\nu} \pi^+) = 2 \Gamma(n \rightarrow \bar{\nu} \pi^0) \end{cases}$$

CALCULATING p-DECAY



$\Rightarrow \alpha_{GUT}, M_V$

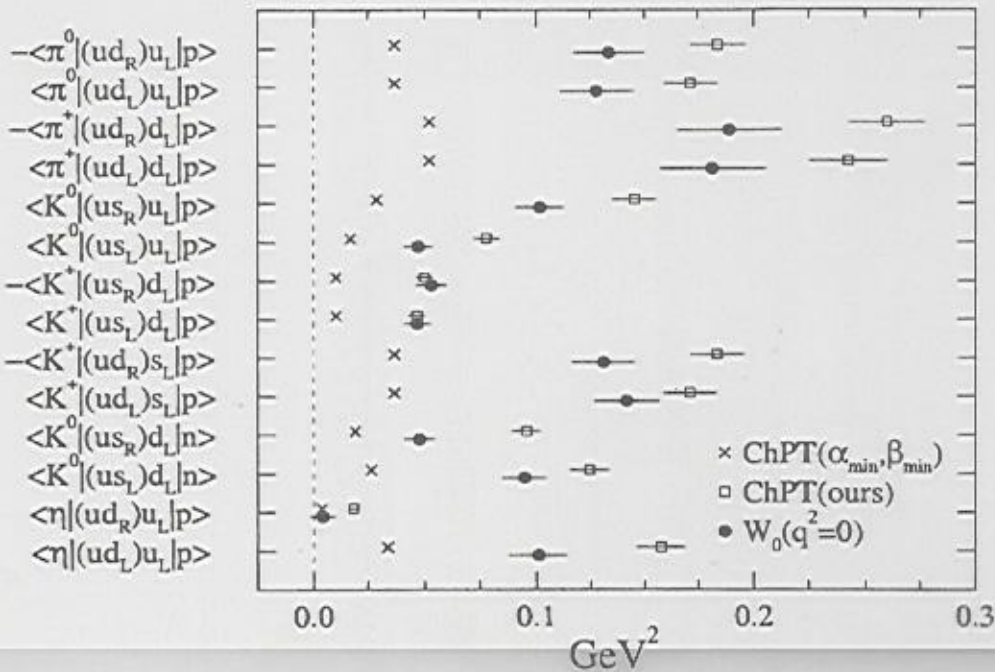
\Downarrow anomalous dimensions

HADRONS $\langle p | O | X \rangle$ matrix element

$|\alpha| = |\beta| = 0.003 \text{ GeV}^3 @ 1 \text{ GeV}$

$|\alpha| = 0.015 \text{ GeV}^3 @ 2.3 \text{ GeV}$ JLQCD
 $|\beta| = 0.014 \text{ GeV}^3 @ 2.3 \text{ GeV}$ JLQCD

almost factor of 20 enhancement

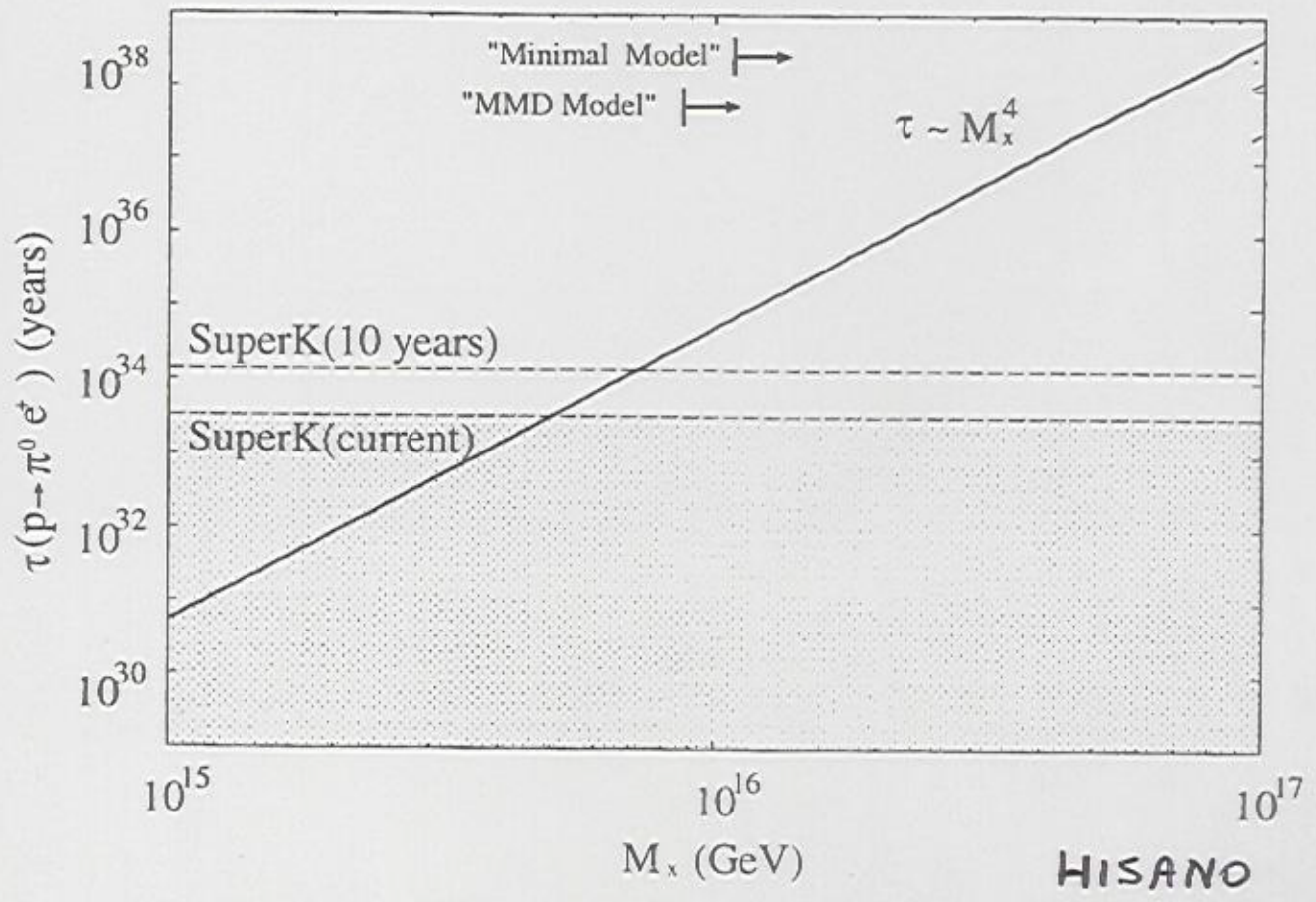


JLQCD Coll.

Using SUSY gauge coupling unification

$$\tau_p(p \rightarrow \pi^0 e^+) = 10^{35} \text{ yrs} \left(\frac{0.015 \text{ GeV}^3}{\alpha} \right)^2 \times$$

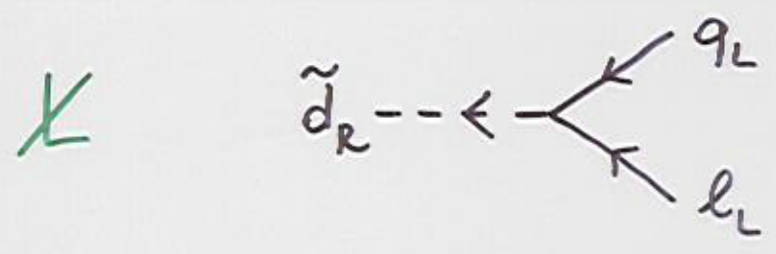
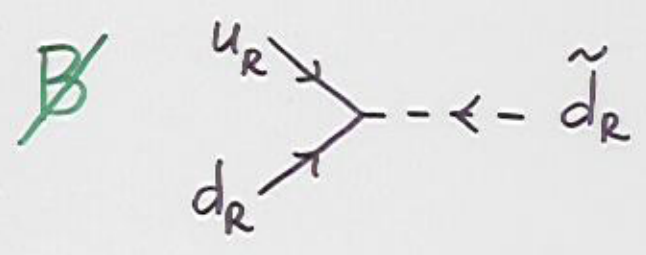
$$\times \left(\frac{M_x}{10^{16} \text{ GeV}} \right)^4 \left(\frac{1/25}{\alpha_{\text{GUT}}} \right)^2$$



SUPERSYMMETRY

Scalars in effective theory
⇒ operators $d < 6$

$d=4$



$$\tau_p \sim 10^{-10} \text{ s} \left(\frac{\tilde{m}}{\text{TeV}} \right)^4 \frac{1}{\lambda^4}$$

R-parity (consequence of gauge symmetry?)

d = 5

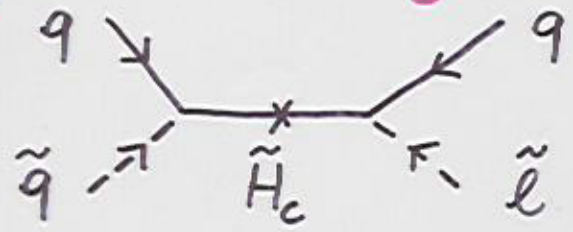
$$W = -\tilde{c}_L \tilde{O}_L - \tilde{c}_R \tilde{O}_R$$

$$\tilde{O}_L = Q_L^k Q_L^l Q_L^i L_L^j$$

$$\tilde{O}_R = \bar{U}_R^i \bar{D}_R^j \bar{U}_R^k \bar{E}_R^l$$

- \tilde{O}_L vanishes if $k=l=i$
- \tilde{O}_R vanishes if $i=k$

Generated by



$$\tilde{c}_L^{ijkl} = \frac{1}{2M_{H_c}} (\lambda_D)^{ij} (V^T P \lambda_u V)^{kl}$$

$$\tilde{c}_R^{ijkl} = \frac{1}{M_{H_c}} (P^* V^* \lambda_D)^{ij} (\lambda_u V)^{kl}$$

- Yukawa coupling (with naive SU_5 mass relations)

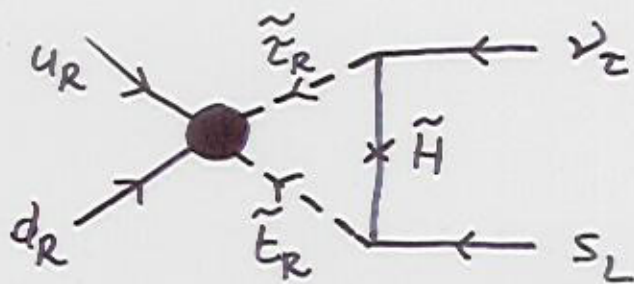
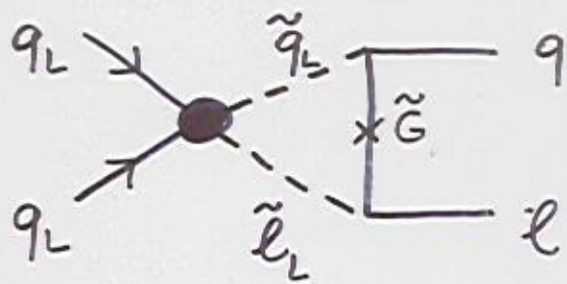
• M_{H_c}

• Two new phases

$$P = \begin{pmatrix} e^{i\phi_{13}} & & \\ & e^{i\phi_{23}} & \\ & & 1 \end{pmatrix}$$

DRESSING

(16)



$p \rightarrow K^+ \bar{\nu}$ dominates over

$p \rightarrow \pi^+ \bar{\nu}$ Cabibbo-suppressed

$p \rightarrow K^0 \mu^+$ suppressed by m_u

Rate depends upon

- susy mass spectrum
- flavour violation in susy sector
- couplings and mass of H_c
- new phases

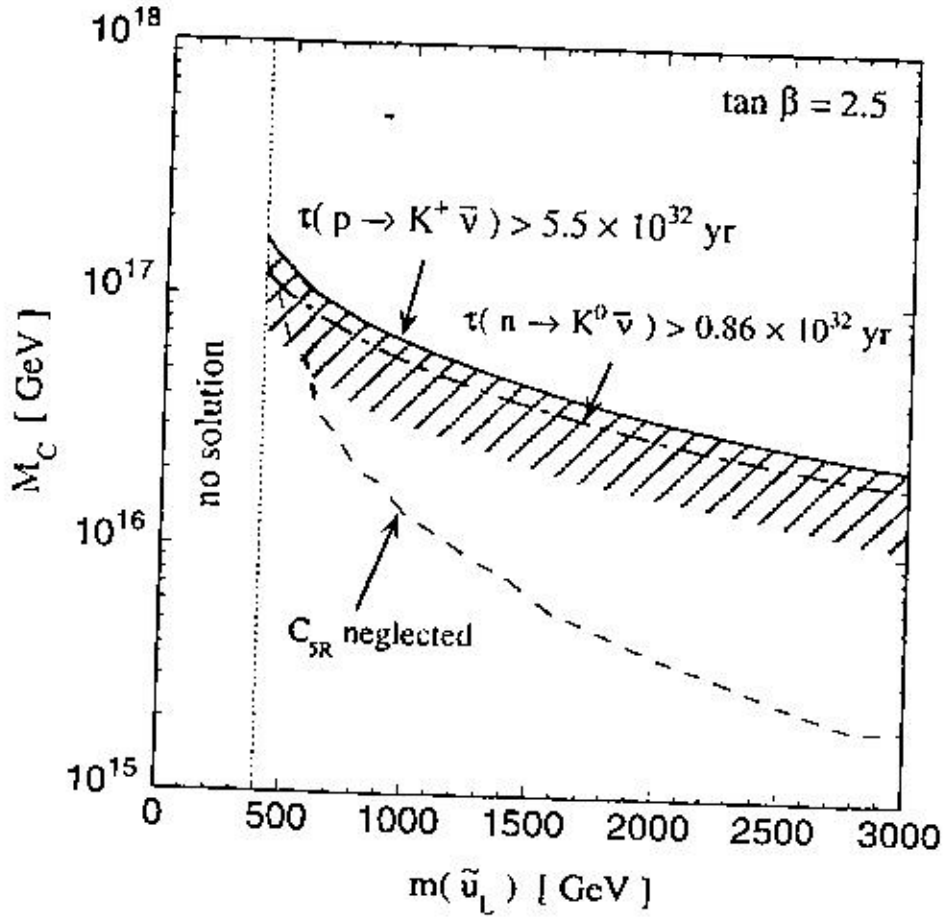
$$\mathcal{A}(p \rightarrow K^+ \bar{\nu}_e) = e^{i\phi_{23}} A_e(\tilde{c}_L) + A_e(\tilde{t}_L)$$

$$\mathcal{A}(p \rightarrow K^+ \bar{\nu}_\mu) = e^{i\phi_{23}} A_\mu(\tilde{c}_L) + A_\mu(\tilde{t}_L)$$

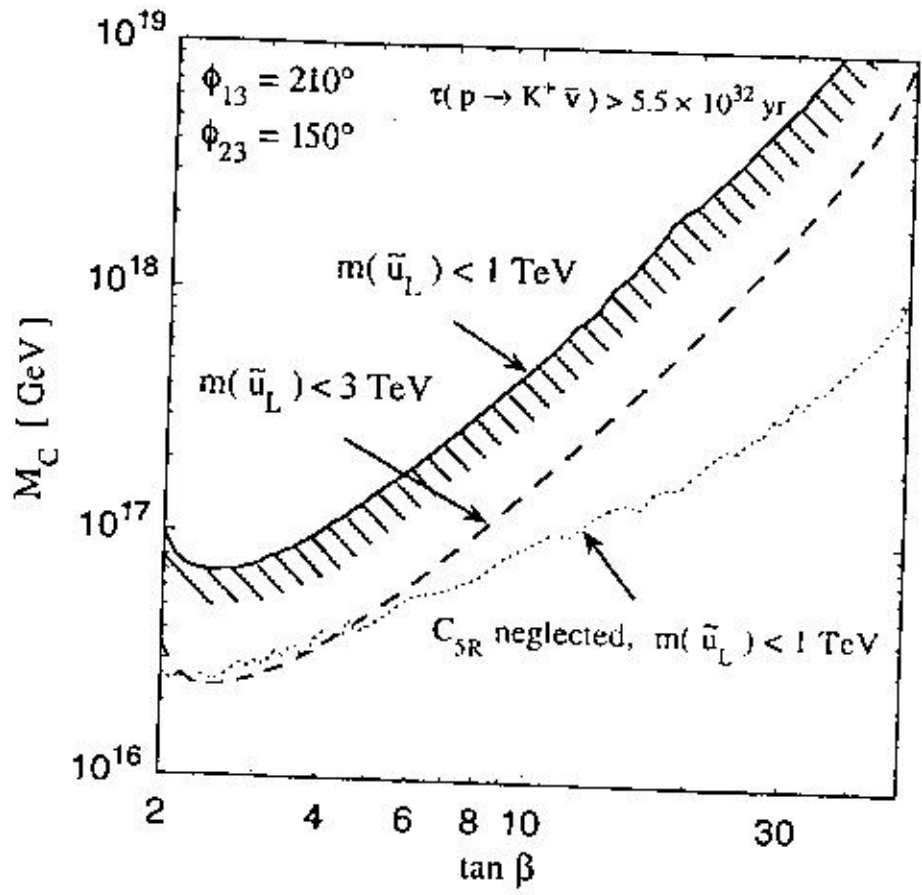
$$\mathcal{A}(p \rightarrow K^+ \bar{\nu}_\tau) = e^{i\phi_{23}} A_\tau(\tilde{c}_L) + A_\tau(\tilde{t}_L) + e^{i\phi_{13}} B(\tilde{t}_R)$$

ϕ_{23} such that $\mathcal{A}(p \rightarrow K^+ \bar{\nu}_{e,\mu}) \approx 0 \Rightarrow \mathcal{A}(p \rightarrow K^+ \bar{\nu}_\tau)$ large

ϕ_{13} " $p \rightarrow K^+ \bar{\nu}_\tau$ " $p \rightarrow K^+ \bar{\nu}_\mu$ "



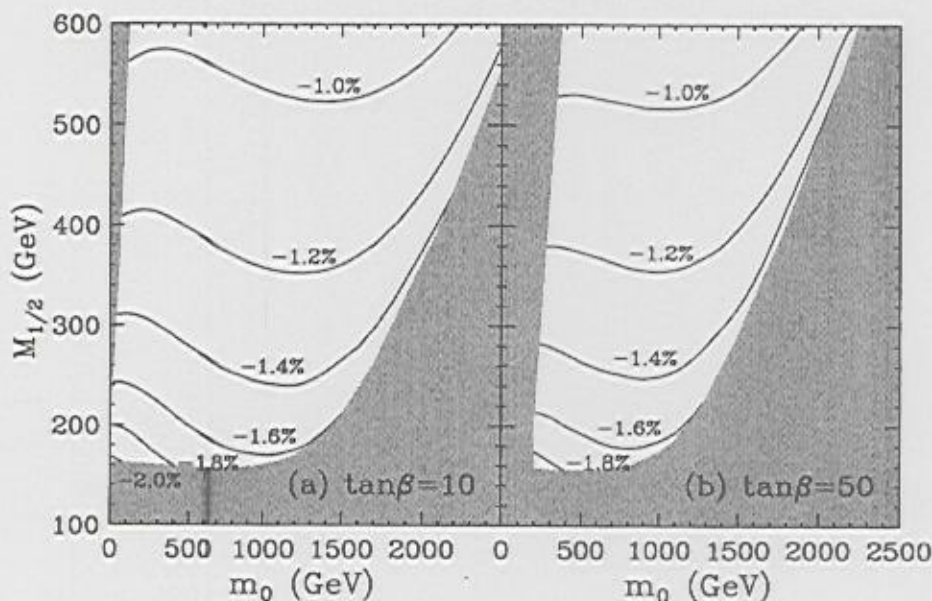
GOTO-NIHEI



Determining M_{H_c} from threshold corrections

Define $g_1(M_{GUT}) = g_2(M_{GUT})$

$$\epsilon \equiv \frac{g_3(M_{GUT}) - g_1(M_{GUT})}{g_1(M_{GUT})}$$



FENG
MATCHEV

$$\delta \alpha_s \approx 0.7 \epsilon$$

$$\epsilon_{H_c} = 0.3 \frac{\alpha_{GUT}}{\pi} \ln \left(\frac{M_{H_c}}{M_{GUT}} \right)$$

$$\Rightarrow 3.5 \times 10^{14} < M_{H_c} < 3.6 \times 10^{15} \text{ GeV}$$

(90% CL)

- Thresholds from other GUT particles?

$d=5$ PROTON DECAY

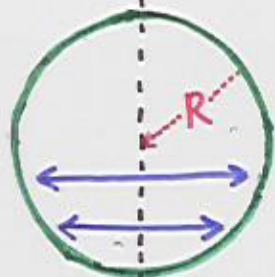
- depends on unknown aspects of SUSY GUT
 - doublet-triplet splitting
 - fermion mass relations

most plausible estimate
in conflict with observations

- mechanism to suppress or eliminate $d=5$ operators

GUT in $D > 4$

Compactification geometry eliminates some states



$$S_1 / \mathbb{Z}_2$$

identify opposite points

5-dim field \Leftrightarrow tower of 4-dim fields

For fields with definite parity

$$\phi(x, y) \rightarrow \phi(x, -y) = \pm \phi(x, y)$$

$$\phi_+(x, y) = \sum_{n=0}^{\infty} \phi_+^{(n)}(x) \cos \frac{ny}{R}$$

$$\phi_-(x, y) = \sum_{n=0}^{\infty} \phi_-^{(n)}(x) \sin \frac{ny}{R}$$

ϕ_- has no massless 4-D mode

Theory in 4-D may have smaller gauge group

SU_5 in 5-D

- no H_c in 4-D

- no $d=5$ p-decay

- $d=6$ p-decay may be enhanced

CONCLUSIONS

p-decay fundamental test
of conservation laws

Implications for:

- understanding of
short-distance interactions
- GUTs
- baryogenesis
- matter stability
- ultimate fate of the Universe