

# Transition Radiation by Neutrinos at an interface

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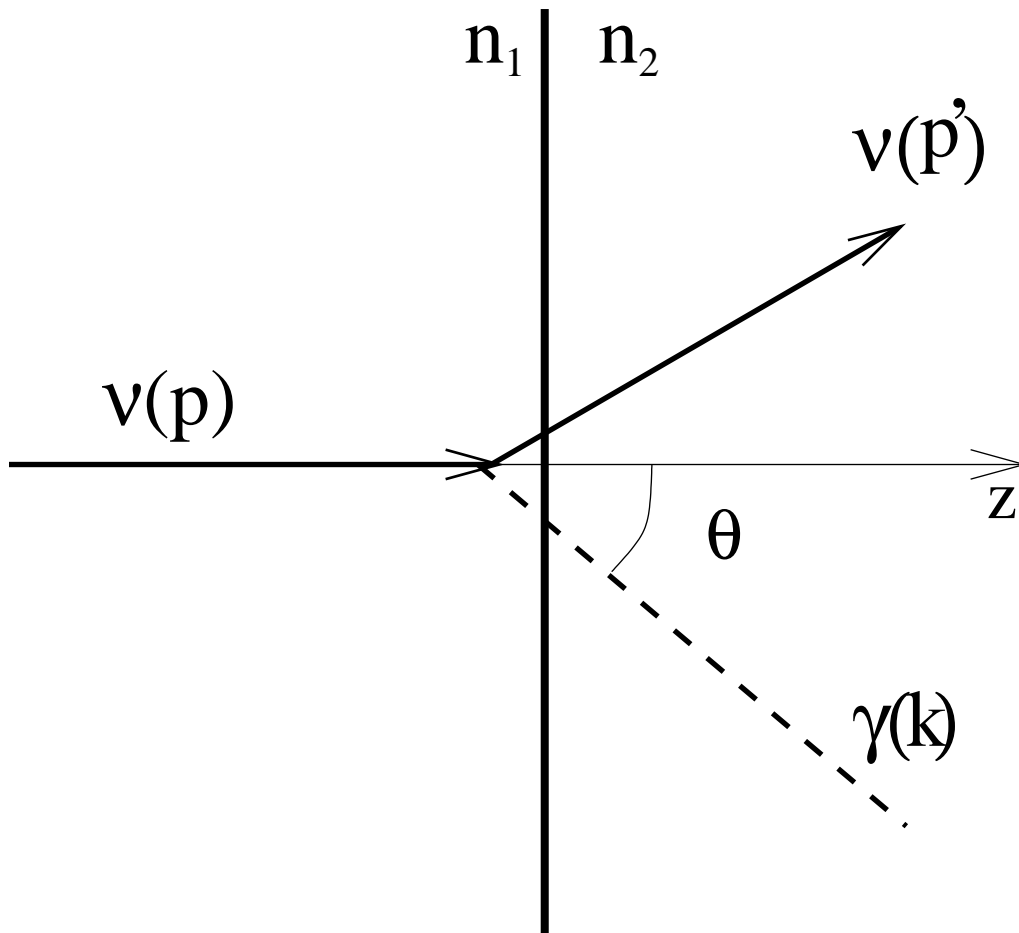


Figure 1: Transition radiation by neutrino at an interface of two media with refractive indexes  $n_1$  and  $n_2$ .

The conservation of momentum is a consequence of translational invariance.

$p \neq p'_z + k_z$  Kinematically allowed.

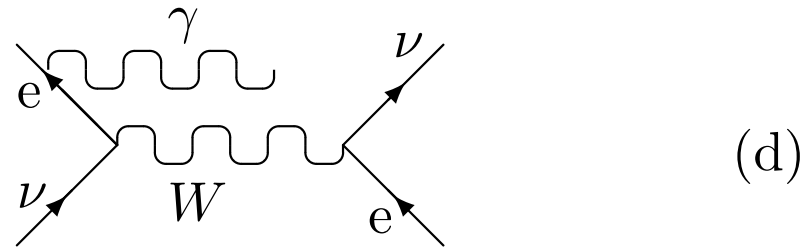
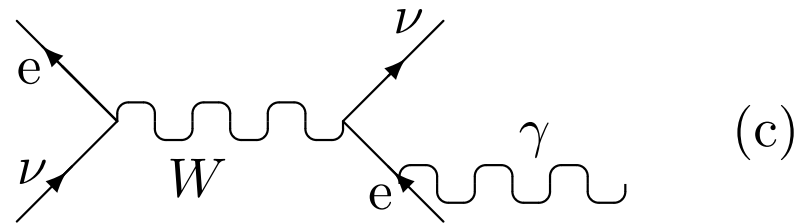
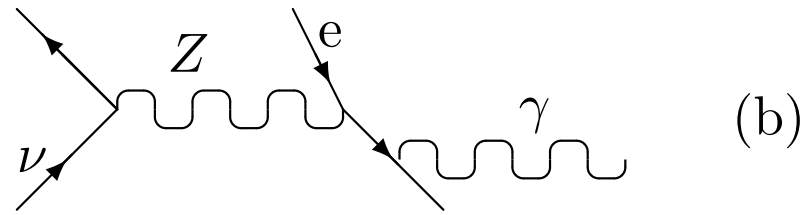
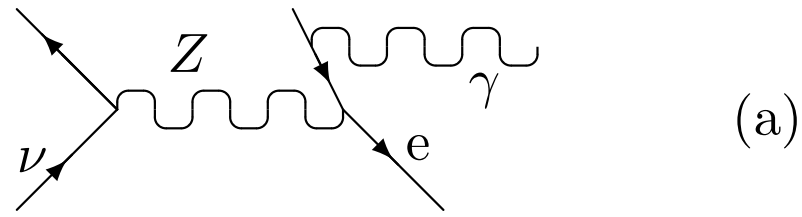


Figure 2: Neutrino-photon coupling in electrons background. (a,b)  $Z$ - $\gamma$ -mixing. (c,d) "Penguin" diagram (only for  $\nu_e$ ).

Grigoriy Garibyan -armenian physicist, discoverer and developer of the theory of X-ray transition radiation. He used both - classical electrodynamics and **quantum electrodynamics** approaches to describe the phenomena.

$$m_\nu \ll E_\nu, \omega, \omega_p$$

Let us consider a neutrino crossing the interface between two media with refraction indices  $n_1$  and  $n_2$  (see fig. 1). In terms of the matrix element  $\mathcal{M}$  the transition radiation probability of the process  $\nu \rightarrow \nu\gamma$  is

$$W_{\nu \rightarrow \nu\gamma} = \frac{1}{(2\pi)^3} \frac{1}{2E\beta_z} \int \frac{d^3\mathbf{p}'}{2E'} \frac{d^3\mathbf{k}}{2\omega} \cdot \delta(E - E' - \omega) \delta(p'_x + k_x) \delta(p'_y + k_y) \sum_{pols} \left| \int_{-\infty}^{\infty} dz e^{i(p_z - p'_z - k_z)z} \mathcal{M}_{\nu \rightarrow \nu\gamma} \right|^2$$

$p = (E, \mathbf{p})$ ,  $p' = (E', \mathbf{p}')$ ,  $k = (\omega, \mathbf{k})$ ,  $\beta_z = p_z/E$ . The sum is over photon polarizations.

We assume  $p^2 = 0$  and  $E = |\mathbf{p}|$ . After integration over  $d\mathbf{p}'$  and  $z$

$$W_{\nu \rightarrow \nu \gamma} = \frac{1}{(2\pi)^3} \frac{1}{8E\beta_z} \int \frac{|\mathbf{k}|^2 d|\mathbf{k}|}{\omega E' \beta'_z} \sin \theta d\theta d\varphi$$

$$\sum_{pols} \left| \frac{\mathcal{M}_{\nu \rightarrow \nu \gamma}^{(1)}}{p_z - p_z^{(1)} - k_z^{(1)}} - \frac{\mathcal{M}_{\nu \rightarrow \nu \gamma}^{(2)}}{p_z - p_z^{(2)} - k_z^{(2)}} \right|^2$$

$\beta'_z = p'_z/E'$ ,  $\theta$  is the angle between the emitted photon and incoming neutrino.  $\mathcal{M}_{\nu \rightarrow \nu \gamma}^{(1,2)}$  are matrix elements in each media.

$$p_z^{(i)} = \sqrt{(E - \omega)^2 - n^{(i)2} \omega^2 + k_z^{(i)2}}.$$

Formation zone length in each media

$$l^{(i)} = (p_z - p_z^{(i)} - k_z^{(i)})^{-1}$$

$$\max(l^{(i)}) = |\omega(1 - n)|^{-1} \approx \frac{2\omega}{\omega_p^2} \approx 1 \text{ cm} \frac{\omega}{\text{MeV}} \left(\frac{20 \text{ eV}}{\omega_p}\right)^2$$

In a media the photon refractive index is  $n = \sqrt{\epsilon\mu}$ .

$$\pi_t = \frac{1}{2} T_{\mu\nu} \pi^{\mu\nu}$$

$$\pi_t = (1 - n_t^2) \omega^2$$

$$T^{\mu\nu} = -g^{\mu i} \left( \delta_{ij} - \frac{\mathbf{k}_i \mathbf{k}_j}{\mathbf{k}^2} \right) g^{j\nu},$$

$$\begin{aligned} M_{\nu \rightarrow \nu \gamma} &= -\frac{G_F}{\sqrt{2}e} Z \epsilon_\mu \bar{u}(p') \gamma_\nu (1 - \gamma_5) u(p) (g_V \pi^{\mu\nu} - g_A \pi_5^{\mu\nu}) \\ &= \frac{G_F}{\sqrt{2}e} Z \epsilon_\mu \bar{u}(p') \gamma_\nu (1 - \gamma_5) u(p) g^{\mu i} \left( g_V \pi_t \left( \delta_{ij} - \frac{\mathbf{k}_i \mathbf{k}_j}{\mathbf{k}^2} \right) - i g_A \pi_5 \epsilon_{ijkl} \frac{\mathbf{k}_l}{|\mathbf{k}|} \right) g^{j\nu} \\ g_V &= \begin{cases} 2 \sin^2 \theta_W + \frac{1}{2} & \text{for } \nu_e \\ 2 \sin^2 \theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases}, \quad g_A = \begin{cases} +\frac{1}{2} & \text{for } \nu_e \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases} \end{aligned}$$

$$\pi_5 \simeq \frac{|\mathbf{k}|}{2m_e} \omega^2 (1 - n_t^2)^2 = \frac{|\mathbf{k}|}{2m_e} \frac{\omega_p^4}{\omega^2}$$

$$\pi_t = \omega_p^2, \quad \omega_p^2 = \frac{4\pi\alpha N_e}{m_e}. \quad \omega_p = 20\text{eV}$$

Let us consider a neutrino crossing the interface between media with refraction indices  $n$  ( $n^2 = 1 - \frac{\omega_p^2}{\omega^2}$ ) and vacuum.  $p'_z = \sqrt{(E - \omega)^2 - n^2 \omega^2 \sin^2 \theta}$

$$W_{\nu \rightarrow \nu \gamma} = \frac{G_F^2}{16\pi^3 \alpha} \int \frac{\omega d\omega \sin \theta d\theta}{(E - p'_z - n\omega \cos \theta)^2} \cdot$$

$$\left[ (g_V^2 \pi_t^2 + g_A^2 \pi_5^2) \left( 1 - \frac{\cos \theta}{E - \omega} (p'_z \cos \theta - n\omega \sin^2 \theta) \right) \right.$$

$$\left. - 2g_V g_A \pi_t \pi_5 \left( \cos \theta - \frac{1}{E - \omega} (p'_z \cos \theta - n\omega \sin^2 \theta) \right) \right]$$

$$\theta \sim \sqrt{1 - n^2}.$$

After integration over angle  $\theta$  we get

$$\begin{aligned} W &\simeq \frac{G_F^2}{16\pi^3\alpha} \int \frac{d\omega}{\omega} \left[ -\ln\left[(1 - n^2)\left(1 - \frac{\omega}{E}\right)\right] \right. \\ &\quad \left. + \ln\left[(1 - n^2)\left(1 - \frac{\omega}{E}\right) + \theta_{max}^2\right] - 1 \right] \\ &\times \left[ (g_V^2\pi_t^2 + g_A^2\pi_5^2)\left(2 - 2\frac{\omega}{E} + \frac{\omega^2}{E^2}\right) - 2g_V g_A \pi_t \pi_5 \frac{\omega}{E} \left(2 - \frac{\omega}{E}\right) \right]. \end{aligned}$$

$$W = \int_{\omega_{min}}^E dW \simeq \frac{g_V^2 G_F^2 \omega_p^4}{16\pi^3\alpha} \left( 2 \ln^2 \frac{E}{\omega_p} - 5 \ln \frac{E}{\omega_p} + \delta \right)$$

here  $\delta \simeq 5$  for  $\omega_{min} = \omega_p$ ,  $\delta \simeq -1$  for  $\omega_{min} = 10\omega_p$ .

The energy deposition of the neutrino in the media due to the transition



radiation

$$\int_{\omega_p}^E \omega dW_{\nu \rightarrow \nu \gamma} \simeq \frac{g_V^2 G_F^2 \omega_p^4}{16\pi^3 \alpha} E \left[ \frac{8}{3} \ln \frac{E}{\omega_p} - 4.9 + 9 \frac{\omega_p}{E} + O\left(\frac{\omega_p^2}{E^2}\right) \right]$$

For MeV electronic neutrinos the transition radiation probability is about  $W \sim 10^{-39}$  and the energy deposit is about  $1.4 \cdot 10^{-34}$  eV when they cross the interface of the media with  $\omega_p = 20$  eV to vacuum.

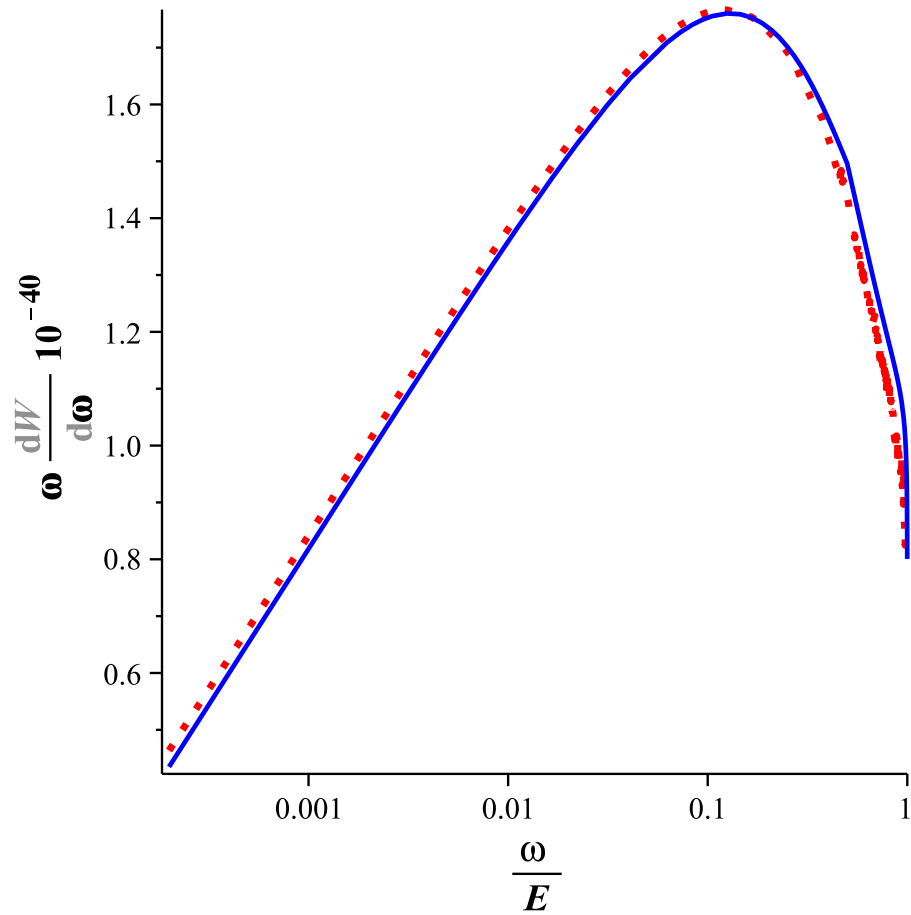


Figure 3: Energy spectrum of the NTR.  $E_\nu = 1$  MeV and  $\omega_p = 20$  eV. Logarithmical scale.

# Summary

We calculate the transition radiation process  $\nu \rightarrow \nu\gamma$  at the border of two media.

The neutrinos are taken to be with only standard-model couplings and we ignore the neutrino masses due to their negligible contribution.

The medium fulfills the dual purpose of inducing an effective neutrino-photon vertex and of modifying the photon dispersion relation.

The process is kinematically allowed. The violation of the translational invariance at the direction from one media into another leads to the momentum non conservation at the same direction.

We find that the probability of transition radiation is larger by three orders of magnitude (medium induced neutrino-photon vertex) than previous calculations (vacuum induced vertex **PL B435 (1998) 134**).

The process may be important for the SuperNovae neutrinos.